COVENANT UNIVERSITY NIGERIA

TUTORIAL KIT OMEGA SEMESTER

PROGRAMME: CHEMICAL ENGINEERING

COURSE: CHE 522

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CHE522: PROCESS OPTIMIZATION

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QUESTION 1

The NIPOST-Post Master general is planning to embark upon the production of two grades of stamps X_1 and X_2 which requires 3 units of labor to manufacture X_1 and 6 units of labour to manufacture X_2, and all he has available are 24 units of labour. Furthermore, it requires 2 units of raw materials to manufacture X_1 and 1 unit to manufacture X_2 and all he has available are 10 units of raw materials. The profit per unit on stamp grades X_1 is ₦2, and on stamp grade X_2 is ₦3. How many units of each product should be made in order to maximize profit?

QUESTION 2

A gas processing plant receives a fixed amount of raw gas each week. The raw gas is processed into two grades of heating gas, regular and premium quality. These grades of gas are in high demand (that is, they are guaranteed to sell) and yield different profits to the company. However, their production involves both time and on-site storage constraints. For example, only one of the grades can be produced at a time and the facility is open for only 80 hrs/week. Further, there is limited on-site storage for each of the products. All these are listed below.

	Product		
Resource	**Regular**	**Premium**	**Resource Availability**
Raw gas (m^3 /tonne)	7	11	77
Production time (hrs/tonne)	10	8	80
Storage (tone)	9	6	
Profit (₦/tonne)	150	175	

Recall that a metric ton, or tonne = 1000kg

Develop a linear programming formulation to maximize the profits for this operation. **Use the graphical approach only.**

QUESTION 3:

A manufacturer makes two types of products 1 and 2. Three machines A, B and C are required for the manufacture of each product. 1 unit of product 1 requires 2 hours on machine A, I hour on machine B, and 6 hours on machine C while 1 unit of product 2 requires respectively 2 hours, 5 hours and 2 hrs on machine A, B and C. In a given period there are 24hrs available on machine A, 44hrs on B and 60hrs on

C. The profit per unit product 1 is N6 and on product 2 is N9. Given that machines are available when required, how many units of each product should be made in order to maximize profit?

QUESTION 4: **Use the graphical method to Maximize** $P = x + 0.7y$

Subject to: $70x + 31y \leq 6000,\ 6x + 9y \leq 2400$ and $24x + 60y \leq 12000.\ x, y \geq 0$

QUESTION 5

Use the Simplex method to maximize the function: $P = x + 4y$

Subject to: $-x + 2y \leq 6$ and $5x + 4y \leq 40.\ x, y \geq 0$

QUESTION 6;

A company manufactures two types of products I and II. It requires 3 units of labour to manufacture product I and 6 units of labour to manufacture product II, and all he has available are 24 units labour. Furthermore, it requires 2 units of raw materials to manufacture product I and 1 unit to manufacture product II and all he has available are 10 units of raw materials. The profit per unit on product I is ₦2 and on product II is ₦3. How many units of each product should be made in order to maximize profit?

Use the Simplex method.

QUESTION 7

(a) Explain what is meant by Queuing Theory and describe the basic structure of Queuing models.

(b) Show and explain how queuing models are conventionally labeled. [**2 marks]**

(c) The county hospital emergency management engineer has concluded that the emergency cases arrive pretty much at random (a Poisson input process), so that inter arrival times have an exponential distribution. She also has concluded that the time spent by a doctor treating the cases approximately follows an exponential distribution. By projecting the available data for the early evening shift into next year, she estimates that patients will arrive at average rate of one for every 6 $minutes$. If the relationship between the arrival and service rates is given by the equation $= \sqrt[3]{19.683\left(1 + \sqrt{(\mu^3)}\right)}$, determine

(i) The percentage of time the doctor will be idle

(ii) The probability that four patients will be in the system

(iii) Average number of patients in the system

(iv) The average queue length

(v) The average time a patient spends in the system

(vi) The average time a patient spends in the queue

(vii) State two conditions that have enabled the use of all the equations you have used so far.

QUESTION 8

6(b) Bank customers arrive at a single drive –in window at an average rate of 4 vehicles in every quarter of an hour. On the average the customers need three minutes each to transact their business. Given that the arrival pattern is described by the Poisson distribution and that the departure time is exponentially distributed, calculate the following:

(i) The percentage of time the bank teller will be idle.

(ii) The probability that five customers will be in the system.

(iii) The Average number of customers in the system.

(iv) The average queue length.

(v) The average time each customer spends in the system.

(vi) The average time each customer spends in the queue

QUESTION 9

(i)(a) Define Network Analysis.

(b) What is the major property difference between the Shortest Path Problem and the Minimum Spanning Tree Problem?

(ii) A park has recently been set aside for a limited amount of sightseeing and backpack hiking. Cars are not allowed into the park, but there is a narrow, winding road system for trains and jeeps drivers by the pack rangers. The road system is shown in Figure 3.1 below.

The numbers give the distances of these winding roads in Kilometres. The park contains a scenic wonder at station T. A small number of trains are used to transport sightseers from the park entrance to station T and back. The park management currently faces three problems: (a) one is to determine which route from the park entrance to station T has the shortest total distance for the operation of the trains. The second problem is that telephone lines must be installed under the roads to establish telephone communication among all the stations (including park entrance). Because the installation is both expensive and disruptive to the natural environment, lines will be installed under just enough roads to provide some connection between every pair of stations. (b) Where should the lines be laid to accomplish this with minimum number of kilometres of line installed? (c) How do you route the various trips to increase the number of trips that can be made per day without violating the limit in any individual road?

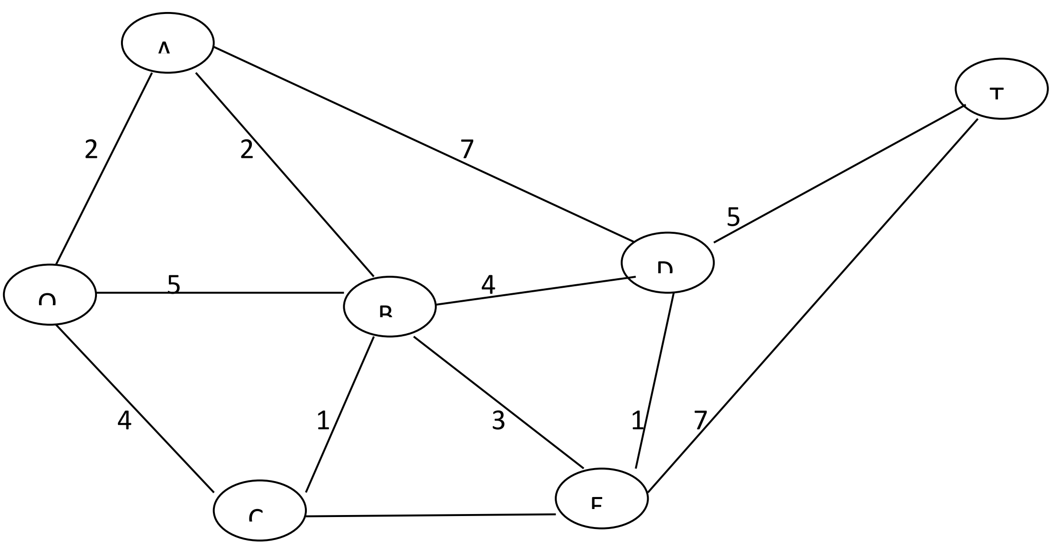

FIGURE 3.1

(10) A shipping company must move 50 units of a product from Los Angeles to New York through other depots. Table 1 below, gives transportation costs in dollars per unit between the company's various depots. The blank entries in the table signify that shipments cannot be made directly between corresponding depots. Find the cheapest shipping schedule. Solve as shortest route problem and show details.**]**

	L. Angeles	**San Francisco**	**Phoenix**	**Laramie**	**St. Louis**	**Chicago**	**New York**
L. Angeles	-	7	8	-	39	-	95
San Francisco	7	-	22	17	-	36	85
Phoenix	8	22	-	14	25	27	-
Laramie	-	17	14	-	31	19	
St. Louis	39	-	25	31	-	14	20
Chicago	-	36	27	19	14	-	13
New York	95	85	-	-	20	13	-

Q11. With illustration, distinguish between a unimodal function and a multimodal function.

Q12. Briefly explain the significance of optimization of chemical processes.

Q13. Consider the objective function: $J(\underline{x}) = 2x_1^2 + 2x_1x_2 + 1.5x_2^2 + 7x_1 + 8x_2 + 24$.

(i) Determine the optimal solution with classification.
(ii) By using the eigenvalues in the analysis, determine the convexity or concavity of the function.
(iii) Find the equations for the principal axes and determine a transformation $\underline{x} = \underline{V}\,\underline{z}$.

Q14(i) Is the function $y(x) = \dfrac{2x-3}{1-e^{1/(2x-3)}}$ continuous? Where are the discontinuities in the first derivative?

(ii) Define typical regions in which the following functions: $y = \sin x$; $y = x^2$; $y = e^x$; and $y = x$; are found to be (a) monotonic increasing; (b) monotonic decreasing; (c) unimodal; (d) bimodal; (e) concave and (f) convex.

(iii) Given the function: $y = \begin{cases} -x^2 & -\infty \le x \le 0 \\ -x^2 & 0 \le x \le 1 \\ e^{x-1} & 1 \le x \le \infty \end{cases}$.

- (i) Is the function continuous?
- (ii) Does the function have any continuous derivatives?
- (iii) Is it monotonic?
- (iv) Is the function unimodal?

Q15. Which scheme of two experiments at a time and three experiments at a time is more efficient in determining the optimal solution to the function: $J(x) = (2x - 9)^2 \big| 0 \le x \le 1$. Hence, find $x^* = \min\{J(x) = (2x - 9)^2 \big| 0 \le x \le 1\}$ by reducing the original interval to 5% of its original value.

Q16. 4,000 kg/h of a 5% by wt solution of ethanal in toluene is to be treated with water to extract ethanal in a multi-staged countercurrent solvent extraction process. If the annual capital and operating costs per stage are estimated to be ₦32 million, the cost of supplying, pumping and regenerating the solvent from the final extract is estimated to be ₦80/kg, and the value of ethanal is taken as ₦276/kg, what is the optimum number of stages and quantity of solvent required for the process? The distribution ratio of ethanal in water to ethanal in toluene is considered constant at 2.2.

Q17(i) The first three experiments of an unconstrained search for a maximum yield the following results:

Experiment	x	$J(x)$
1	1	83
2	2	69
3	4	39

Determine the estimate of the scale of the objective function using direct search method. [**14 marks**]

(ii) Consider the objective function: $J(\underline{x}) = x_1^2 + x_2^2$, subject to:

$$h_1(\underline{x}) = x_2 + x_1 - 6 \le 0$$

$$h_2(\underline{x}) = x_2 - x_1 - 2 \le 0$$

$$h_3(\underline{x}) = x_1 - 2x_2 - 2 \le 0$$

By introduction of appropriate slack variables, obtain optimal solution at the vertex of the intersection of $h_1(\underline{x})$ and $h_3(\underline{x})$ using the method of Lagrange multiplier. [**34 marks**]

Q18. Minimize $J(x) = e^{2x} + 1.5x^2$, using Newton's and quasi-Newton methods, with the initial guess, $x_0 = 1$, and the change in $x \leq 10^{-4}$. Use $h = 10^{-2}$ for the quasi-Newton method. Tabulate your results for comparison.

Q19(i) Using direct substitution method, Find $\underline{x}^* = \min \ or \ \max\left\{J(\underline{x}) = x_1^2 + x_2^2 \middle| \underline{g}(\underline{x}) = x_1 + x_2^2 - 5 = 0\right\}$

(ii) Due to scale formation (fouling), the overall heat transfer coefficient in a batch evaporation operation decreases with time according to: $\frac{1}{U^2} = 0.5t + 5$. Consequently, the evaporator must be cleaned from time to time for a period of 2.5 hours. Determine the total boiling time in 1-day of operation that maximizes the heat transferred within these hours.

Q20. Find $x^* = \min \ or \ \max\left\{J(x) = x^2 + 2e^{-x} \middle| D = \{x : 0 \leq x \leq 1\}\right\}$ by using methods of (i) Quadratic interpolation; (ii) Cubic interpolation; (iii) Newton-Gregory interpolating polynomial; and (iv) Lagrange interpolating polynomial. Compare your results and draw inferences.