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TUTORIAL KIT
OMEGA SEMESTER

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COURSE: GEC 220

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GEC 220: ENGINEERING MATHEMATICS 11

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SECTION A

Given that $3\frac{d^2y}{dx^2} + 15\frac{dy}{dx} + 18y = 2x$ **(use this to answer questions 1-4**)

1. The roots of the auxiliary equation are ……………………………….…. (**Show your workings)** . 2. The complimentary function is ……………………………………………

3. The Particular integral (PI) is ……………………………………………

4. The general solution (GS) is………………………………………………

5. The general formula for finding integrating is I.F = (a) $e^{\int I.Fpdx}$ (b) $e^{\int Qdx}$ (c) $(b)e^{ln\int Pdx}$ (d) $e^{\int Pdx}$

Given that the third order determinant A= $\begin{bmatrix} 3 & -1 & -4 \\ 1 & 2 & -13 \\ 2 & -5 & 11 \end{bmatrix}$**(use this to answer questions 6-8**)

6. $|A|$ is ……………………………………………………………………… (**Show your workings)**

7. What is the co-factor of element A_{13}? ……………………………………… (**Show your workings)**

8. What is A^T? …………………………………………………

Given that A= $\begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$ **(use this to answer questions 9-10**)

9. Product AB is ………………………………………. **Show your workings where possible)**

10. Product BA is ……………………………………… **Show your workings where possible**

11. Given a matrix A, if $AA^T = I$ then A is said to be (a) an Identity matrix (b) Unit Matrix (c) orthogonal (d) a product of the matrix A and transpose of A.

12. The $\int \frac{sinx}{cosx}dx$ = (a) –lnsinx (b) –lncosx (c) lncosx (d) lnsinx (e) none of the above

13. What is e^{-2lnx}? (a) lnx (b) x^2 (c) x^{-2} (e) $\frac{1}{x}$ (e) $\frac{1}{2}x$

14 What is $e^{-lnx^{-1}}$? (a) –lnx (b) –x (c) $\frac{1}{x}$ (d) x

Consider the differential equation: $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 5y = 4e^{2x}$ **(use this to answer questions 15 – 17)**

15 The assumed solution is (a) $-4Ce^{-2x}$ (b) $Ce^{2x} + D$ (c) Ce^{-2x} (d) $Ce^{-2x} + Dx + E$

16 $M_1 = ?, M_2 = ?$ (**show your workings)**

17 The general solution is $y =$ (a) $Ae^y + Be^{5y}$ (b) $Ae^{-x} + Be^{-5x}$ (c) $Ae^x + Bxe^{5x}$ (d) $Ae^{-y} + Bxe^{-5y}$

18 solve the equation $\frac{dy}{dx} = e^{3x-2y}$ given that when x $= 0, y = 0$ (a) $e^{2y} = \frac{2}{3}e^{3x} + \frac{1}{3}$ (b)$e^{3y} = \frac{2}{3}e^{2x} + \frac{C}{3}$

(c) $y = \frac{2}{3}e^{3x-2y} + \frac{1}{3}$ (d) $e^{2y} = \frac{3}{2}e^{3x} + \frac{C}{3}$ (e) $e^{3y} = \frac{2}{3}e^{2x} + \frac{1}{3}$ **show your workings (3marks)**

Given that $-\frac{dy}{dt} \propto xy$, if $k = 0.5$ and the value of $x$at any time is x $= \frac{4}{(1+t)^2}$ and initially, $y = 10$, Use this to answer questions 19-21 (**show your workings)**

19 Write down the value of the integration constant

20. Then y as a function of t is (a) $y = \frac{1}{(1+t)} + 0.303$ (b) $y = \frac{1}{(1+t)} + 0.503$ (c) $lny = \frac{2}{(1+t)} + 0.503$

(d) $lny = \frac{2}{(1+t)} + 0.303$ (e) non of the above

21. When t → ∞ (a) $y = 0.503$ (b) $y = 1.35$ (c) $y = 0.303$ (d) $y = 1.53$

Consider an RLC circuit with $R = 10ohms, C = 0.1farad, E(t) = 30e^{-3t}$.

22. From Kirchhoff's voltage law, write the resultant first order differential equation.

23. Solve the first order differential equation in 11 (**show your workings)**

(a) $I(t) = \frac{9}{2}e^{-3t}+Ce^{-t}$ (b) $I(t) = \frac{2}{9}e^{-3t}+Ce^{-3t}$ (C) $I(t) = \frac{9}{2}e^{-t}+e^{-3t}$ (d) $I(t) = \frac{2}{9}e^{-t}+e^{-3t}$

24.Given that $I(0) = 0$, what is the value of the constant C for the correct option in No 12

(a) $C = -\frac{9}{2}$ (b) $C = -\frac{2}{9}$ (C) $C = \frac{9}{2}$ (d) $C = \frac{2}{9}$ (d) none of the above

25. if $R = 10\ ohms, C = 0.1\ farad, L = 2\ henrys\ and\ E(t) = 30e^{-3t}$then the circuit is

(a) Critically damped (b) Under damped (c) Over damped (d) Zero damping (**2marks) Show your workings**

For 26-21 If the steady state equation of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = -85cos3x$is

$y = 7cos3x - 6sin3x,$

26. Find the stationary value (when $\frac{dy}{dx} = 0$) of the steady state equation. **Show your workings**

(a) $6tan3x = -7$ (b)) $7tan3x = -6$ (c)) $6Cos3x = -7$ (d)) $7Cos3x = -6$

27. Write the steady state equation in the form of $KSin\ (3x + \theta)$ **Show your workings or use formulae**

(a) $9.2Sin\ (3x - 49.4)$ (b) $9.5Sin\ (3x + 49.4)$ (c) $9.2Sin\ (3x + 45)$ (d) $9.5Sin\ (3x - 45)$

28. What are the respective values of the frequency and period of the steady state equation?

(a) $\frac{3}{\pi}$ & $\frac{\pi}{3}$ (b) $\frac{3}{\pi}$ & $\frac{2\pi}{3}$ (c) $\frac{3}{2\pi}$ & $\frac{2\pi}{3}$ (d) 9.5 & 3 (e) 9.2 & 3

29. Write the general form of an exact differential equation ____________

30. What is the necessary and sufficient condition for a differential equation to be exact?

(a) $\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$ (b) $\frac{\delta M}{\delta x} = \frac{\delta N}{\delta y}$ (c) $U = \int Mdx + k(y)$ (d) No condition is met

SECTION B (THEORY)

1 During a fermentation process the rate of decomposition of a substance at any time t varies directly as the amount of substance y and also as the amount of active ferment x. If the constant of proportionality is 0.5, the value of x at any time t is $\frac{4}{(1+t)^2}$ and initially y = 10.

(i) find y as a function of t.

(ii) Deduce the amount of substance remaining as t becomes very large

2 Use the substitution method $y = \frac{v}{x}$ where v is a function of x only, to transform the equation $\frac{dy}{dx} + \frac{y}{x} = xy^2$ into a differential equation in V and x. Hence find y in terms of x. 3. Find the current in an RLC –circuit with $R = 3\ ohms, L = 0.5\ henry,$

$C = 0.08\ farad, E(t) = 12cos5t$ assuming zero initial current and charge.

(b) Find the amplitude, period and frequency of the steady state current (c) What sort of response do you expect from the circuit?

4. Solve the equation $0.5\frac{d^2x}{dt^2} - \frac{3}{2}cost + 2x = 0$ given that

$x = 0\ at\ t = 0\ and\ that\ \frac{dx}{dt} = -1\ at\ t = \frac{\pi}{2}$. Find the maximum value of x in the interval $0 < t < \pi$

5. The volume *V* of a liquid of viscosity coefficient *η* delivered after time *t* when passed through a tube of length *L* and diameter *d* by a pressure *p* is given by $V = \frac{pd^4t}{128\eta L}$. If the errors in *V*, *p*and *L* are 1%, 2% and 3% respectively, ***using total differential,*** determine the error in *η*. Assume there in no error in *t* and *d.*

6. A rectangular box has sides of length *x* cm, *y* cm and *z* cm. Sides *x* and *z* are expanding at rates of 0.3 cm/s and 0.5 cm/s respectively and side *y* is contracting at a rate of 0.2 cm/s. Using chain rule, determine the rate of change of volume when *x* is 3 cm, *y* is 1.5 cm and *z* is 6 cm.

7. A possible equation of state for a gas takes the form:

$PV = RT\, e^{\left(\frac{-\alpha}{VRT}\right)}$in which$\alpha$ and *R* are constants. Using the ***general implicit***formula, determine expressions for $\frac{\partial P}{\partial V}, \frac{\partial V}{\partial T}\ and\ \frac{\partial T}{\partial P}$

8. The equation $3 = z^3+3xz$ defines z implicitly as a function of x and y. Using the **general implicit formula** evaluate, $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

9. Determine and classify the critical points of the function: $f(x,y) = x^3 + xy^2 - 12x - y^2$

10. Locate and determine the nature of the stationary points on the surface

$f\,(x,y) = 2x^3+2y^3-6x-24y+16$

11 Find the first three non-zero terms in the Maclaurin series of the function: e^{sinx}.

12. Compute the 4th order Taylor series of $\cos x$ about $x = \pi/3$

GEC 220: ANSWERS

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SOLUTIONS TO SECTION A:

1: $M_1 = -2$, $M_2 = -3$

3. $y = \frac{1}{9}x + \frac{5}{54}$

5. $I.F = e^{\int Pdx}$

7. Co-factor is = -7

9. AB is not possible. The reason is that the number of columns in A is not equal to the number of rows in B.

11 Othogonal

13. x^{-2}

15. Ce^{2x}

17. No answer.

19. $C = -0.303$

21. $y = 1.35$

23. $I(t) = \frac{9}{2}e^{-3t} + Ce^{-t}$

25. Over-damped

27. $9.2\sin(3x - 49.4$

29. $M(x, y)dx + N(x, y)dy = 0$

SOLUTIONS TO SECTION B (THEORY):

Q1.

$$-\frac{dy}{dt} \propto yx$$

$$-\frac{dy}{dt} = kyx \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.. (1)$$

Given that $k = 0.5, eqn\ 1\ then\ becomes$

$$-\frac{dy}{dt} = 0.5yx \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

Given also that x= $\frac{4}{(1+t)^2}, eqn\ 2\ becomes$

$$-\frac{dy}{dt} = 0.5X\frac{4}{(1+t)^2}y$$

$$\frac{dy}{dt} = -\frac{2y}{(1+t)^2}$$

$$and\ separating\ variables\ gives$$

$$-\frac{dy}{y} = \frac{2}{(1+t)^2}dt$$

$$and\ integrating\ gives$$

$$\int\frac{dy}{y} = -\int\frac{2}{(1+t)^2}dt$$

$$\int\frac{dy}{y} = -2\int(1+t)^{-2}\,dt$$

$$lny = -2\frac{(1+t)^{-1}}{-1} + C$$

$$lny = \frac{2}{1+t} + C \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. (3)$$

$$but\ at\ t = 0, y = 10$$

$$eqn\ 3\ becomes$$

$$ln10 = \frac{2}{1+0} + C = 2 + C$$

$$\therefore C = ln10 - 2 = 2.303 - 2 = 0.303$$

$$hence\ eqn\ 3\ becomes$$

(i) $lny = \frac{2}{1+t} + 0.303 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$

(ii)

$$as\ t\ becomes\ very\ large,$$

$$t \to \infty$$

$$eqn\ 4\ becomes$$

$$lny = \frac{2}{1+\infty} + 0.303$$

$$lny = 0 + 0.303$$

$$y = e^{0.303}$$

$$y = 1.354$$

Q3.

$$From\ kirchoff\ voltage\ law,$$

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i(t)dt = E(t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$differentiating\ eqn\ 1 gives$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = \frac{dE(t)}{dt} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$where\ \frac{dE(t)}{dt} = -60sin5t$$

$$and\ substituting\ known\ values\ in\ 2\ gives$$

$$0.5\frac{d^2i}{dt^2} + 3\frac{di}{dt} + \frac{1}{0{,}08}i = -60sin5t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (3)$$

$$= \frac{d^2i}{dt^2} + 6\frac{di}{dt} + 25i = -120sin5t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (4)$$

$$solving\ the\ 2nd\ order\ differential\ eqn\ gives$$

$$\frac{d^2i}{dt^2} + 6\frac{di}{dt} + 25i = 0$$

$$M^2 + 6M + 25 = 0$$

$$solving\ quadratically\ gives\ that\ M = -3 \pm j4$$

$$It\ is\ a\ complex\ root. hence\ \alpha = -3\ and\ \beta = 4$$

$$For\ complex\ number, the\ complimentary\ solution\ is\ obtained\ from$$

$$i = e^{\alpha t}(cos\beta t + sin\beta t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (5)$$

$$Substituting\ known\ values\ gives\ the\ CF:\ i = e^{-3t}(cos4t + sin4t)$$

$$and\ solving\ the\ RHS\ of\ eqn\ 4\ gives\ the\ particular\ integral\ (PI)$$

$$given\ f(t) = -120sin5t$$

$$the\ assumed\ solution\ becomes$$

$$i = Ccos5t + Dsin5t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (i)$$

$$\frac{di}{dt} = -5Csin5t + 5Dcos5t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots ... (ii)$$

$$\frac{d^2i}{dt^2} = -25Ccos5t - 25Dsin5t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (iii)$$

$$substituting\ eqns\ (i - iii)\ in\ eqn\ 4\ gives$$

$$-25Ccos5t - 25Dsin5t + 6(-5Csin5t + 5Dcos5t) + 25Ccos5t + 25Dsin5t = -120sin5t$$

$$-25Ccos5t - 25Dsin5t + (-30Csin5t + 30Dcos5t) + 25Ccos5t + 25Dsin5t = -120sin5t$$

$$collecting\ like\ terms\ gives$$

$$(-25C + 30D + 25C)cos5t + (-25D - 30C + 25D)sin5t = -120sin5t$$

$$30Dcos5t - 30Csin5t = -120sin5t$$

$$and\ comparing\ co - efficient\ gives\ that$$

$$-30C = -120 \Rightarrow C = 4$$

$D = 0$

$$hence\ from\ eqn\ i,$$

$$i = 4cos5t \qquad (this\ is\ the\ PI.which\ is\ also\ called\ the\ steady\ state\ current)$$

$$hence\ the\ general\ solution\ is\ i = CF + PI$$

$$i = e^{-3t}(Acos4t + Bsin4t) + 4cos5t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (6)$$

$$initially, t = 0, i = 0$$

$$hence\ from\ eqn\ 6,$$

$$0 = (A + 0) + 4, and\ A = -4$$

$$differentiating\ eqn\ 6\ gives$$

$$\frac{di}{dt} = -3e^{-3t}(Acos4t + Bsin4t) + e^{-3t}(-4Asin4t + 4Bcos4t) - 20sin5t \ldots\ldots\ldots (7)$$

$$we\ could\ rewrite\ eqn\ 1\ as$$

$$L\frac{di}{dt} + Ri + \frac{1}{C}Q = E(t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots. (8)$$

$$at\ t = 0, i = 0. Q = 0, hence\ from\ eqn\ 8$$

$$0.5\frac{di}{dt} + 0 + 0 = 12cos5t = 12$$

$$\frac{di}{dt} = 24$$

$$and\ substituting\ for\ \frac{di}{dt}\ in\ eqn\ 7\ gives$$

$$24 = -3(-4) + 1(0 + 4B)$$

$$24 - 12 = 4B\ and\ B = 3$$

$$substituting\ for\ A\ and\ B\ in\ eqn\ 6\ gives$$

$$i = e^{-3t}(3sin4t - 4cos4t) + 4cos5t$$

Find the current in an RLC –circuit with $R = 3\ ohms, L = 0.5\ henry,$

$C = 0.08\ farad, E(t) = 12cos5t$ assuming zero initial current and charge.

(b) Find the amplitude, period and frequency of the steady state current (c) What sort of response do you expect from the circuit?

(b) $The\ steady\ state\ current\ is$

$$i = 4cos5t$$

$$the\ amplitude\ K = \sqrt{C^2 + D^2} = \sqrt{4^2 + 0^2 = 4}$$

$frequency, f = \frac{5}{2\pi}\ and\ period\ p = \frac{1}{f} = \frac{2\pi}{5}$

(ii) The sort of response expected from this type of circit may be obtained from

$$R^2 = 3^2 = 9$$

$$\frac{4L}{C} = \frac{4\ X\ 0.5}{0.08} = 25$$

$$since\ R^2 < \frac{4L}{C},$$

$$The\ system\ is\ underdamped.$$

Q5.

$$V = \frac{pd^4t}{128\eta L}$$

$V = f(p, d, t, \eta, L)$

Using total differential

$dV = \frac{\partial V}{\partial P}dp + \frac{\partial V}{\partial d}dd + \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial \eta}d\eta + \frac{\partial V}{\partial L}dL$

$\frac{\partial V}{\partial P} = \frac{d^4t}{128\eta L}$

$\frac{\partial V}{\partial d} = \frac{4pd^3t}{128\eta L}$

$\frac{\partial V}{\partial t} = \frac{pd^4}{128\eta L}$

$\frac{\partial V}{\partial \eta} = -\frac{pd^4 t}{128\eta^2 L}$

$\frac{\partial V}{\partial L} = -\frac{pd^4 t}{128\eta L^2}$

$dV = 0.01V$

$dP = 0.02P$

$dL = 0.03L$

$dd = dt = 0$

Hence, the total differential equation becomes:

$$0.01V = \frac{d^4 t}{128\eta L} 0.02P + \frac{4pd^3 t}{128\eta L}(0) + \frac{pd^4}{128\eta L}(0) - \frac{pd^4 t}{128\eta^2 L}\, d\eta - \frac{pd^4 t}{128\eta L^2}\, 0.03L$$

$0.01V = 0.02\frac{pd^4 t}{128\eta L} + (0) + \ (0) - \frac{1}{\eta}\frac{pd^4 t}{128\eta L}\, d\eta - 0.03\frac{pd^4 t}{128\eta L}$

But $V = \frac{pd^4 t}{128\eta L}$

$\therefore\ 0.01V = 0.02V - \frac{d\eta}{\eta} V\ - 0.03V$

Thus, $\frac{d\eta}{\eta} = 0.02$

$d\eta = 0.02\,\eta = \frac{2}{100}\eta$

$\therefore$ The % error in η is 2%

Q7.

$\therefore PV - RT\, e^{\left(\frac{-\alpha}{VRT}\right)} = 0$

Using the general implicit formula,

$\frac{\partial P}{\partial V} = -\frac{F_V}{F_P}$

$\frac{\partial V}{\partial T} = -\frac{F_T}{F_V}$

$\frac{\partial T}{\partial P} = -\frac{F_P}{F_T}$

$$F_P = \frac{\partial\left[PV - RT\, e^{\left(\frac{-\alpha}{VRT}\right)}\right]}{\partial P}$$

$= V$

$$F_V = \frac{\partial\left[PV - RT\, e^{\left(\frac{-\alpha}{VRT}\right)}\right]}{\partial V}$$

$$= P - RT\frac{\partial\left[e^{\left[\left(\frac{-\alpha}{RT}\right)V^{-1}\right]}\right]}{\partial V}$$

$$= P - RT\left[\frac{\alpha}{V^2RT}e^{\left(\frac{-\alpha}{VRT}\right)}\right]$$

$$= P - \frac{\alpha}{V^2}e^{\left(\frac{-\alpha}{VRT}\right)}$$

$$F_T = \frac{\partial\left[PV - RT\, e^{\left(\frac{-\alpha}{VRT}\right)}\right]}{\partial T}$$

$$= 0 - \left[RT\frac{\partial\left[e^{\left[\left(\frac{-\alpha}{VR}\right)T^{-1}\right]}\right]}{\partial T} + e^{\left(\frac{-\alpha}{VRT}\right)}\frac{\partial\,[RT]}{\partial T}\right]$$

$$= -\left[RT \cdot \frac{\alpha}{VRT^2}e^{\left(\frac{-\alpha}{VRT}\right)} + R\cdot e^{\left(\frac{-\alpha}{VRT}\right)}\right]$$

$$= -R\cdot e^{\left(\frac{-\alpha}{VRT}\right)}\left[\frac{\alpha}{VRT}+1\right]$$

$$\therefore \boldsymbol{\frac{\partial P}{\partial V} = -\frac{F_V}{F_P}}$$

$$\boldsymbol{= -\frac{P-\frac{\alpha}{V^2}e^{\left(\frac{-\alpha}{VRT}\right)}}{V}}$$

$$\frac{\partial V}{\partial T} = -\frac{F_T}{F_V} = -\frac{-R\cdot e^{\left(\frac{-\alpha}{VRT}\right)}\left[\frac{\alpha}{VRT}+1\right]}{P - \frac{\alpha}{V^2}e^{\left(\frac{-\alpha}{VRT}\right)}}$$

$$\boldsymbol{= \frac{R\cdot e^{\left(\frac{-\alpha}{VRT}\right)}\left[\frac{\alpha}{VRT}+1\right]}{P-\frac{\alpha}{V^2}e^{\left(\frac{-\alpha}{VRT}\right)}}}$$

$$\frac{\partial T}{\partial P} = -\frac{F_P}{F_T} = -\frac{V}{-R\cdot e^{\left(\frac{-\alpha}{VRT}\right)}\left[\frac{\alpha}{VRT}+1\right]}$$

$$\boldsymbol{= \frac{V}{R\cdot e^{\left(\frac{-\alpha}{VRT}\right)}\left[\frac{\alpha}{VRT}+1\right]}}$$

Q9

$$f(x,y) = x^3 + xy^2 - 12x - y^2$$

At critical points, for a function of $f(x,y)$, $f_x = f_y = 0$

$f_x = 3x^2 + y^2 - 12 = 0$ ------------------------ (1)

$f_y = 2xy - 2y = 0$ ------------------------ (2)

From Equation (2),

$2y(x-1) = 0$

Thus, $\boldsymbol{y = 0}$ **or** $\boldsymbol{x = 1}$ ------------------------- (3)

Substitute Equation (3) into Equation (1)

<u>When y = 0, Equation (1) becomes:</u>

$$3x^2 - 12 = 0$$

Therefore,
$x = \pm 2$ i.e $x = +2$ or $x = -2$ ---------------------- (4)

Hence, critical points occur at the points:

$$\boldsymbol{(x, y) = (2, 0) \;\; and \;\; (-2, 0)}$$

<u>When x = 1, Equation (1) becomes:</u>

$$3 + y^2 - 12 = 0$$

Therefore,
$x = \pm 3$ i.e $x = +3$ or $x = -3$ ---------------------- (5)

Hence, other critical points occur at the points:

$$\boldsymbol{(x, y) = (3, 0) \;\; and \;\; (-3, 0)}$$

To classify the points, we use second derivative test:

$\boldsymbol{D_{(a,b)} = f_{xx}(a,b).f_{yy}(a,b) - \left[f_{xy}(a,b)\right]^2}$

$$f_x = 3x^2 + y^2 - 12$$

$\therefore f_{xx} = 6x$

and$f_{xy} = 2y$

$$f_y = 2xy - 2y$$

$\therefore f_{yy} = 2x - 2$

$$\therefore D_{(a,b)} = 6x\,.(2x-2) - \;(2y)^2$$

$$= 12x^2 - 12x - \;4y^2$$

$\boldsymbol{= 12\,x(x-1) - \;4y^2}$

At$\underline{\boldsymbol{(x, y) = (2, 0)}}$

$$\therefore D_{(2,0)} = 12\,(2)(2-1) - \;4(0)^2$$

=24

Thus, $D_{(2,0)}$ is positive

$$\therefore f_{xx}(2{,}0) = 6(2)$$

= 12

$\therefore f_{xx}(2{,}0)$is also positive

Therefore, critical point $(\boldsymbol{x}, \boldsymbol{y}) = (\mathbf{2}, \mathbf{0})$is a **MINIMUM POINT**

At$\underline{(\boldsymbol{x}, \boldsymbol{y}) = (-\mathbf{2}, \mathbf{0})}$

$$\therefore D_{(-2,0)} = 12\,(-2)(-2-1) - \,4(0)^2$$

=72

Thus, $D_{(-2,0)}$ is positive

$$\therefore f_{xx}(-2{,}0) = 6(-2)$$

$= \ -12$

$\therefore f_{xx}(-2{,}0)$is negative

Therefore, critical point $(\boldsymbol{x}, \boldsymbol{y}) = (-\mathbf{2}, \mathbf{0})$is a **MAXIMUM POINT**

At$\underline{(\boldsymbol{x}, \boldsymbol{y}) = (\mathbf{1}, \mathbf{3})}$

$$\therefore D_{(1,3)} = 12\,(1)(1-1) - \,4(3)^2$$

$= \ -36$

Thus, $D_{(1,3)}$ is NEGATIVE

Therefore, critical point $(\boldsymbol{x}, \boldsymbol{y}) = (\mathbf{1}, \mathbf{3})$is a **SADDLE POINT**

At$\underline{(\boldsymbol{x}, \boldsymbol{y}) = (\mathbf{1}, -\mathbf{3})}$

$$\therefore D_{(1,3)} = 12\,(1)(1-1) - \,4(-3)^2$$

$= \ -36$

Thus, $D_{(-1,3)}$ is NEGATIVE

Therefore, critical point $(\boldsymbol{x}, \boldsymbol{y}) = (-\mathbf{1}, \mathbf{3})$is a **SADDLE POINT**

Q 11

For Maclaurin series,

$f(x) \cong f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$f(x) = \ e^{sinx}$ $\qquad$ $f(0) = \ e^{sin0}$ $\qquad$ =1

$f'(x) = cosx \ \cdot \ e^{sinx}$ $\qquad$ $f'(x) = cos0 \ \cdot \ e^{sin0}$ $\qquad$ = 1

$f''(x) = cosx \cdot cosx\, e^{sinx} + e^{sinx} \cdot -sinx$

$\therefore f''(0) = cos0 \cdot cos0\, e^{sin0} + e^{sin0} \cdot -sin0$ = 1

Substitute these into the Maclaurin series equation

$\therefore \boldsymbol{e^{sinx} \cong 1 + x + \frac{1}{2}x^2}$