COVENANT UNIVERSITY
NIGERIA

TUTORIAL KIT
OMEGA SEMESTER

PROGRAMME: MECHANICAL ENGINEERING

COURSE: MCE 326

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MCE326: Fluid Dynamics

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Q1
What is fluid kinematic?
Q2
What is the different between Langrangian and Eulerian method?
Q3
What is the Langrangian method of describing fluid motion?
Q4
What is a path line in fluid motion? What happens to path line during steady and unsteady flows?
Q5
Explain the following: (i) the value of velocity at right angle to the stream line (ii) can there be any flow across the-stream line and (3) can two stream lines cross each other?

Q6
What do you understand by streak line or filament line?
Q7
Distinguish between stream lines, streak lines and path lines
Q8
What is a potential line?
Q9
What are different types of displacements of fluid particles?
Q10
Derive the continuity equation in two dimensions in polar coordinates for incompressible fluids.

Q11
Derive the continuity equation in three-dimensional in cartesian coordinates
Q12
What do you understand by rotation and vorticity of fluid particles?
Q13
What are the conditions for irrotational flow?
Q14
What is circulation? How is it related to vorticity?
Q15
Explain the physical significance and use of the term 'stream function'
Q16
Explain the mathematical concept of stream function.
Q17
Show that stream function satisfies the equation or continuity.
Q18
Derive Laplace equation of stream function for irrotation flow.

Q19
Enumerate the properties of stream function ψ
Q20
Derive the Laplace equation for the velocity potential.

MODEL ANSWERS

A1.
Fluid kinematics is a branch of fluid mechanics which deals with the study of velocity and acceleration of the particles of flu ids in motion and their distribution in space without considering any force or energy involved. Kinematics provides - (i) an .idea about the rate of flow which is also called discharge (ii) an idea about different types of velocities of flow.

A3

In Langrangian method, the motion of each particle is defined with respect to its location in space and time from a fixed position before the start of the motion. The movement of single particle is observed which gives the path followed by the particle. The position of the fluid particle in space (x, y, z) at any time from its fixed position (a, b, c) at $t = 0$, shall be

$$x = f_1(a, b, c, t)$$
$$y = f_2(a, b, c, t)$$
$$z = f_3(a, b, c, t)$$

From the position, velocities, u, v and w and acceleration a_x, a_y and a_z can be given as –

(1) *Velocities* $u = \frac{\partial x}{\partial t}, v = \frac{\partial y}{\partial t}$ & $w = \frac{\partial x}{\partial t}$

(2) *Accelerations* $a_x = \frac{\partial^2 x}{\partial t^2}, a_y = \frac{\partial^2 y}{\partial t^2}$ & $a_z = \frac{\partial^2 x}{\partial t^2}$

The resultant of velocities and acceleration can be calculated. The resultant of pressure and densities can also be worked out.

A5

Since velocity of the fluid particle at any point on the stream line is tangential to the stream line, there cannot by any component of velocity normal or right angle to the stream line i.e. the component of velocity at right angle to the stream line is zero.

There cannot be any flow across the stream line as the flow is always tangential to the stream line

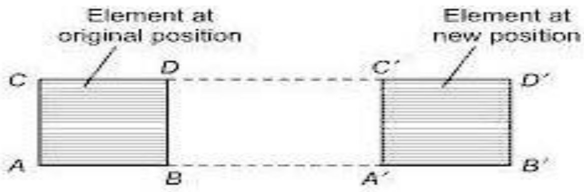
Two stream lines cannot cross each other as otherwise there would be two velocities at that point, one each tangential to the stream lines. This is inconsistent with the definition of a stream line.

A7

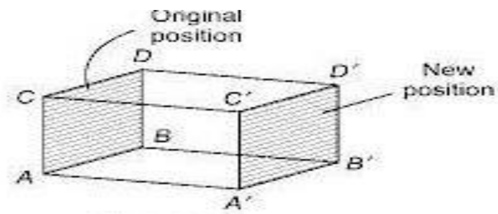
Stream lines	*Streak lines*	*Path lines*
1. Imaginary lines showing positions of various fluid particles	1. Real line showing instantaneous positions of various particles	1. Real line showing successive position of one particle.
2. Particle may change stream line depending on type of flow	2. May change from instant to instant	2. Particle may cross its path line
3. Stream lines cannot intersect each other, they are always parallel	3. Streak line changes with time. Two streak lines may intersect each other	3. Two path lines for two particles may intersect each other
4. No flow across stream line	4. Flow across streak line is possible	4. Flow across a path line is possible by other particles.

A9

Any fluid element can be subjected to translation, rotation or distortion during its course of motion. In translation the fluid element moves to another position in the same direction. For example, translation takes place when liquid flows through pipes or in open channel. Pure translation does not cause any stress in the element.

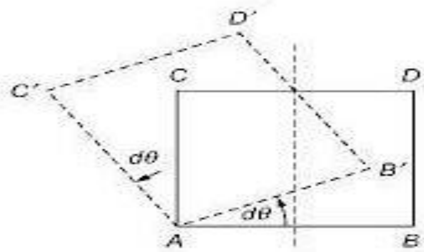


Translation

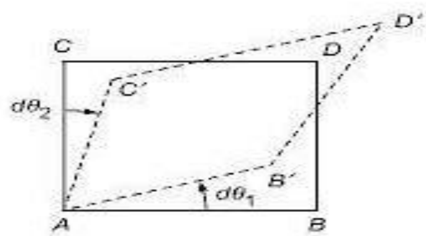


Translation

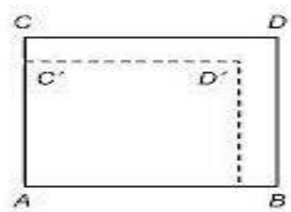
In rotational displacement, the fluid element is subjected to the rotation as shown in the figure. No stress is caused as there is no relative movement of the element with respect to the rotating system. AB and AC are rotating in the same direction to AB' and AC'. Angular deformation is zero $\frac{d\theta - d\theta}{2} = 0$ and shear stress is zero as strain rate $= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = 0$



Distortion can be (1) angular or (2) volume distortion as shown in the figure. Stresses are produced when fluid element is distorted. In angular distortion, stress induced is shear stress due to shear strain as given by the angle of distortion. Angular distortion is $\frac{d\theta_1 + d\theta_2}{2}$ and shear strain rate $= \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial x}\right)$.



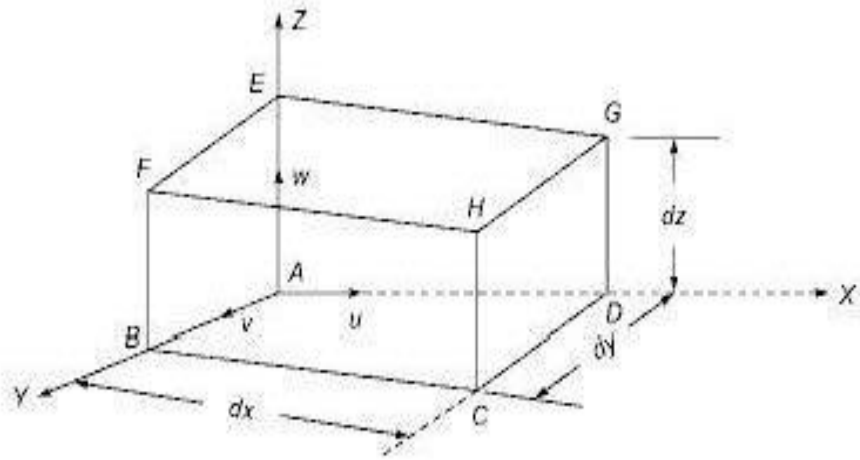
Angular distortion



Volume distortion

A11

Derivation is based on the principle of conservation of mass which states that the quantity of fluid per second remains constant when fluid flows through any section of the pipe.



Fluid Element

Consider a fluid element as shown in the figure above, having lengths dx, dy and dz in x, y and z direction. Assume u, v and w are inlet velocities along x, y and z direction respectively. ρ is the density of fluid element at a particular instant.

Mass of fluid entering from face *ABEF*

$$= \rho \times u \times (dy \times dz)$$

Mass of fluid leaving from face *CDGH*

$$= (\rho u\, dy\, dz) + \frac{\partial}{\partial x}(\rho u \cdot dy\, dz)\, dx$$

Rate of mass increase in x-direction $= \left[(\rho u \cdot dy\, dz) + \frac{\partial}{\partial x}(\rho u \cdot dy\, dz)\, dx\right] - (\rho u\, dy \cdot dz)$

$$= \frac{\partial}{\partial x}(\rho u)\, dx\, dy\, dz$$

Similarly, we can find for y and z direction as under

Rate in mass increase in y-direction $= \frac{\partial}{\partial y}(\rho v) \cdot dx\, dy\, dz$

Rate in mass increase in z-direction $= \frac{\partial}{\partial z}(\rho w)\, dx\, dy\, dz$

Total rate in mass increase $= dx \cdot dy \cdot dz\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w)\right]$

As there is no accumulation of fluid mass, there is no mass increase as per the law of conservation of mass.

Hence $$\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] dx \cdot dy \cdot dz = 0$$

or $$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

If $\rho = 0$ for incompressible fluids

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

This is continuity equation for three-dimensional flow.

For two-dimensional flow, velocity variation along z-direction is zero i.e. $\frac{\partial w}{\partial z} = 0$

$\therefore\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ is continuity equation in two-dimensional flow.

A13

The irrotational flow will have both rotation and vorticity as zero.

$$w = 0 \quad \text{and} \quad \Omega = 0$$

$$\text{or} \qquad w_x = w_y = w_z = 0, \quad \text{and} \quad \Omega_x = \Omega_y = \Omega_z = 0$$

The above gives the conditions of irrotation of the flow as:

$$\frac{\partial w}{\partial y} = \frac{\partial v}{\partial z}$$

$$\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}$$

$$\frac{\partial w}{\partial x} = \frac{\partial u}{\partial y}$$

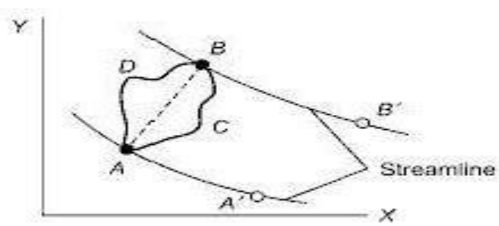
Note: The conditions in polar coordinates is $\Omega = 0$ which gives $\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} + \frac{1}{r}\frac{\partial V_r}{\partial \theta} = 0$

A15

The flow rate (q) between two stream lines per unit thickness and time is known as stream function (ψ).

Physical significance of stream function.

Consider a flow between two stream lines AA' and BB' which is two-dimensional steady and incompressible. The flow (q) per unit time and thickness between these stream lines AA' and BB' is stream function ψ. The flow across points A and B will remain same $= \psi = q$ irrespective of their joining line which may be AB or ACB or ADB. If we assume $\psi = 0$ at point A, then ψ at point B will be flow per unit time and thickness $= q = \psi$



Physical concept of Y'

Stream function is also defined as the scalar function of space and time whose partial derivative with respect to any direction gives the velocity component at right angles to that direction. Hence in steady two-dimensional flow, $\psi = f(x, y)$ which gives $\frac{\partial \psi}{\partial x} = v$ and $\frac{\partial \psi}{\partial y} = u$

A17

The equation of continuity for a two-dimensional flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

As per stream function, we have

$$u = -\frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \psi}{\partial x}$$

Put the values of u and v in the continuity equation (eqn. (1))

$$\frac{\partial}{\partial x}\frac{-\partial \psi}{\partial y} + \frac{\partial}{\partial y}\left(+\frac{\partial \psi}{\partial x}\right) = 0$$

or $$-\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial y \partial x} = 0$$

The above is satisfying as LHS = RHS. Hence stream function satisfies the equation of continuity.

A19

The properties of stream function are:

(1) ψ is constant at all places at a stream line.

(2) The flow around any path in the fluid is zero

(3) The velocity vector at any point on the stream line can be found out by differentiating its stream function i.e. $u = -\frac{\partial \psi}{\partial y}$ and $v = \frac{\partial \psi}{\partial x}$

(4) The discharge between the stream lines is equal to difference of their stream functions i.e.

$$q = \psi_2 - \psi_1$$

(5) The rate of change of stream function with distance is proportion to the component of velocity normal to that direction

(6) If two stream lines superimposed, then $\frac{\partial \psi_1}{\partial s} + \frac{\partial \psi_2}{\partial s} = \frac{\partial (\psi_1 + \psi_2)}{\partial s}$