

**COVENANT UNIVERSITY
NIGERIA**

*TUTORIAL KIT
OMEGA SEMESTER*

PROGRAMME: ECONOMICS

COURSE: CBS 221

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CBS221
STATISTICS FOR BUSINESS & SOCIAL SCIENCES II
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1. A pair of dice is thrown. If the sum on the two dice is 9, find the probability that one of the dice showed 3.
2. The probability that a contractor will get a plumbing contract is $\frac{2}{3}$, and the probability that he will get an electric contract is $\frac{5}{9}$. If the probability of getting at least one contract is $\frac{4}{5}$, what is the probability that he will get both contracts?
3. A question in CBS 211 is given to two students A and B. The probabilities in favour of A solving the question are 6 to 9 and against B solving the question are 10 to 12. If both A and B attempt solving the question, find the probability of the question being solved.
4. Three groups of children contain respectively 3 girls and 1 boy, 2 girls and 2 boys, and 1 girl and 3 boys. One child is selected at random from each group. Show that the probability that the three selected consist of 1 girl and 2 boys is $\frac{13}{32}$.
5. Ten unbiased coins are tossed simultaneously. Find the probability of obtaining (i) exactly six heads (ii) at least 8 heads (iii) no head (iv) at least 1 head (v) not more than 3 heads (vi) at least 4 heads.
6. A ball contains 3 red and 7 white balls. One ball is drawn at random and in its place a ball of the other colour is put in the box. Now one ball is drawn at random from the box. Find the probability that it is red.
7. There are 12 balls in a bag; 8 red and 4 green. Three balls are drawn successively without replacement. What is the probability that they are alternatively of the same colour?
8. A and B play 12 games of chess, 6 out of which are won by A, 4 are won by B and 2 end in a draw. They agree to play a match consisting of 3 games. Find the probability that (i) A wins all three games (ii) 2 games end in a draw (iii) A and B win alternately (iv) B wins at least game.
9. Find the probability of a 4 turning up at least once in two tosses of a fair die.
10. Bag A contains 4 white and 2 black balls; bag B contains 3 white and 5 black balls. If one ball is drawn from each bag, find the probability that (a) both are white (b) both are black (c) one is white and one is black.
11. If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random, (i) 1 (ii) 0, and (iii) at most 2 bolts will be defective.
12. Find the probability that in a family of 4 children there will be (a) at least 1 boy, and (b) at least 1 boy and 1 girl. Assume that the probability of a male birth is $\frac{1}{2}$.
13. The probability that a student that enters the university will graduate is 0.4. Determine the probability that out of 5 students: (i) none will graduate (ii) 1 will graduate (iii) at least 1 will graduate (iv) all will graduate.
14. In a final examination in statistics, the mean was 72 and the standard deviation was 15. Determine the standard scores of students receiving the grades (i) 60 (ii) 93 (iii) 72.
15. Find the probability of getting between 3 and 6 heads inclusive in 10 tosses of a fair coin using the binomial distribution.

16. Ten percent of the tools produced in a certain manufacturing process turn out to be defective. Find the probability that in a sample of 10 tools chosen at random exactly 2 will be defective using (i) the binomial distribution and (ii) the Poisson approximation to the binomial distribution.
17. What is the probability of getting a total of 9 (i) twice and (ii) at least twice in 6 tosses of a pair of dice?
18. An examination in ECN211 was given to Marketing and Industrial Mathematics programmes consisting of 40 and 50 students respectively. In the Marketing class, the mean grade was 74 with a standard deviation of 8, while in the Industrial Mathematics; the mean grade was 78 with a standard deviation of 7. Is there a significant difference between the performance of the two programmes at the (i) 0.05 and (ii) 0.01 levels of significance?
19. The mean height of 50 male students who showed above-average participation in sports was 68.2 inches with a standard deviation of 2.5 inches, while 50 male students who showed no interest in sports had a mean height of 67.5 inches with a standard deviation of 2.8 inches. Test the hypothesis at $\alpha = 0.01$ and 0.05 that male students who participate in sports are taller than other male students.
20. The following are the results of the measurements on each of 10 randomly selected members of a population whose distribution is normal with unknown mean μ and unknown variance σ^2 : 12, 6, 8, 7, 4, 10, 3, 11, 5, 9. Test the hypothesis $H_0: \mu = 30$ against $H_1: \mu \neq 30$ at 95% and 99% levels of confidence.

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ANSWERS

1. (i) Type II Error: Committed when you accept what you should reject (i.e. AR- error). When customer accept and buy fake products they have committed type II error. It is called Consumer Risk. For a good test the probability of committing type 1 error must be small. The probability of committing a type 2 error is called β – risk.

(ii) Level of Significance: This is also called Criterion of Significance. This is the maximum probability of risking a type 1 error. This probability, denoted by α , is often specified before drawing samples.

(iii) Test Statistic: A test statistic (TS) is a function of sample data or a formula. The numerical value obtain when substitutions are made into formula determines the acceptance or rejection of the null hypothesis. The test statistics, being a function of sample data varies from sample to sample. Each test statistics has its own sampling distributions.

(iv) Type 1 Error: Committed when you reject what you should accept (RA- error) when customers reject a good product they have committed type 1 error and the producer loses. It is therefore called producer's risk. The probability of committing type 1 error on a true hypothesis is called α - risk.

(v) Power of a Test: The power of a test, denoted by $1-\beta$, is the probability of correctly rejecting H_0 when it is false.

(vi) Null Hypothesis: The null hypothesis, denoted by H_0 , is the hypothesis of primary interest since it is the one we reject or accept.

3. (a) Time series analysis involves the use of mathematically and statistical methods to determine the behaviour of a given variable over time - past, present and future (**2 mks**).

Usefulness to Social Scientists (**1 mark each**)

- i. It can be use in planning for future operation.
- ii. It is used for the assessment of present achievements.
- iii. It is also used for cross-data comparisons.

3b. (i.) **Secular Variation**

This implies long-term trend. It represents a steady movement or general tendency or pattern over some long period of time. Example population, production level, and income levels etc. They could be linear or nonlinear.

ii. **Seasonal Variation**

They have definite patterns at regular time periods, due mostly to natural phenomena, example, climate, weather conditions, tradition, habit etc.

(iii) **Cyclical Variation:** These are data period movements which have recurrent upward and downward patterns around a reference line such as a secular trend. These trends have no fixed duration's example business cycles.

(iv) **Irregular Variation:** Data points which are sporadic, erratic, random. Irregular variations arise from two basic types of fluctuation- episodic and residual. Examples of episodic - wars, labour strikes, student demonstration; Examples of residual - accident, political election, solar eclipse.

5. a. $H_0: \mu_1 = \mu_2; H_1: \mu_1 \neq \mu_2$

$\alpha = 0.01$ and 0.05

Pooled

$$\text{variance} = \sqrt{\frac{\sigma_1^2}{N_1}} + \sqrt{\frac{\sigma_2^2}{N_2}} = \sqrt{\frac{8^2}{40}} + \sqrt{\frac{7^2}{50}} = 1.606$$

$$z = \frac{74 - 78}{1.606} = |-2.49| = 2.49$$

Since this is two-tailed, we divide the level of significance by 2, at $\alpha = 0.01, \alpha = \frac{0.01}{2} = 0.005$ [$0.5 - 0.005$] = 0.495; at $\alpha = 0.05, \alpha = \frac{0.05}{2} = 0.025$ [$0.5 - 0.025$] = 0.475

At $\alpha = 0.01, z_{tab} = 2.58$, since $z_{tabulated} > z_{calculated}$, we do not reject H_0 , we conclude that there is no significant difference in the performance between the two programmes. At $\alpha = 0.05, z_{tab} = 1.96$, since $z_{tabulated} < z_{calculated}$, we reject H_0 , we conclude that there is a significant difference in the performance between the two programmes and that Business Administration is probably better.

7.

Students	Age (months)	Minimum Stopping at 40 kph (metres)	Age Rank	Stopping Rank	D	D ²
A	9	28.4	1	1	0	0
B	15	29.3	2	2	0	0
C	24	37.6	3	7	-4	16
D	30	36.2	4	4.5	-0.5	0.25
E	38	36.5	5	6	-1	1
F	46	35.3	6	3	3	9
G	53	36.2	7	4.5	2.5	6.25
H	60	44.1	8	8	0	0
I	64	44.8	9	9	0	0
J	76	47.2	10	10	0	0
					$\Sigma =$	32.5

$$\begin{aligned}
 R &= 1 - \frac{6 \Sigma D^2}{N(N^2 - 1)} \\
 &= 1 - \frac{6 * 32.5}{10(10^2 - 1)} \\
 &= 1 - \frac{195}{990} \\
 &= 0.80
 \end{aligned}$$

a. There is a strong correlation between car ages and stopping distance performance.

b. Similarities

- i) They are both measures of degree of association of bivariate data.
- ii) Their values range between -1 and +1.
- iii) Conclusions made about their results are identical.

Differences

- i) They differ in their mathematical formula.
- ii) The Spearman's coefficient is easier to compute than the Pearson's coefficient.
- iii) The Spearman's correlation makes use of ranks while Pearson's correlation makes use of original values in the data.
- iv) The Pearson's correlation is more reliable since it makes use of original values in the data.

9. (i) The properties of the Poisson approximation to the Binomial are:

- (i) The overall experiment consists of a sequence of n identical experiments called trials.
- (ii) Each trial has exactly two possible outcomes. These outcomes are usually referred to as success and failure i.e p and q .
- (iii) The probability of success (and of failure) does not change from one trial to the next.
- (iv) The trials are independent. This implies that the outcome of one trial does not affect the outcome of any other trial.
- (v) The probability of success p , is small and may not be known
- (vi) N is large and may be unknown ($N \geq 50$)
- (vii) The average number of success $\mu = Np$; λ is known and is not large ($\lambda < 5$).

9. (ii) This is a case of the binomial distribution; $X \sim \text{binomial}(n; p; q)$

Where: $n = 3$; $p = 0.45$ and $q = 0.55$

We need to find $P(X=2)$ or $P(X=3) = P(X=2) + P(X=3)$

Applying the binomial formula, we have:

$$P(m = x) = \binom{n}{x} p^x (1 - p)^{n-x} = \frac{n!}{x! (n - x)!} p^x (1 - p)^{n-x}$$

$$P(m = 2) = \frac{3!}{2!1!} 0.45^2 0.55^1 = 0.334125$$

$$P(m = 3) = \frac{3!}{3!0!} 0.45^3 0.55^0 = 0.091125 \text{ (1mk)}$$

$$P(X=2) + P(X=3) = 0.334125 + 0.091125 = 0.42525$$

11. The question here is: “what is the probability that the number of orders is more than 5?”

Let X be the number of orders today.

This is a case of the Poisson random variable, with parameter λ equal to 3 (the average number of orders in a day).

$$P_0 = \frac{e^{-m} \cdot m^x}{x!}$$

We would like to find the probability of $X > 5$, which (in principle) involves adding up the probability for $X = 6$ to the probability for $X = 7$, and adding this to the probability for $X = 8$, and so on, ad infinitum.

The series in question is an infinite one so we can instead find the probability of $X > 5$, which involves only adding 6 terms (see below).

It is the probability of getting 0 orders, plus the probability of getting one, and so on up to the probability of getting 5 orders.

Such that: $P(X > 5) = 1 - P(X \leq 5)$

$$P(X \leq 5) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$$

$$\begin{aligned} &= \frac{3^0}{0!} e^{-3} + \frac{3^1}{1!} e^{-3} + \frac{3^2}{2!} e^{-3} + \frac{3^3}{3!} e^{-3} + \frac{3^4}{4!} e^{-3} + \frac{3^5}{5!} e^{-3} \\ &= e^{-3} + 3e^{-3} + \frac{9}{2}e^{-3} + \frac{9}{2}e^{-3} + \frac{27}{8}e^{-3} + \frac{81}{40}e^{-3} \end{aligned}$$

$$\cong 0.9158$$

This is the probability of not getting a backlog.

Thus, the probability of getting a backlog today is one minus this probability.

$$P(X > 5) = 1 - 0.9158 = 0.0842$$

Therefore, the probability of experiencing a backlog in the manufacturing plant today is 0.0842

13. $H_0: \mu = 150$; $H_1: \mu > 150$

$\alpha = 0.01$ and 0.05

$$\begin{aligned} \bar{x} &= \frac{11+19+16+21+24+27}{6} = \frac{118}{6} = 19.67 \\ \sum X &= 118; (\sum X)^2 = (118)^2 = 13924 \\ \frac{(\sum X)^2}{n} &= \frac{13924}{6} = 2320.67 \\ \sum X^2 &= 11^2 + 19^2 + 16^2 + 21^2 + 24^2 + 27^2 = 2484 \\ s^2 &= \frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1} = \frac{2484 - 2320.67}{5} = 32.67 \\ s &= \sqrt{32.67} = 5.72 \\ t &= \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{19.67 - 150}{5.72/\sqrt{6}} = \frac{-130.33}{2.33} = |-55.82| = 55.82 \end{aligned}$$

Degree of freedom = $n-1=6-1=5$

It is one tailed, at $\alpha = 0.01 = 3.365$; at $\alpha = 0.05 = 2.015$, therefore, at both levels of confidence, we reject H_0 since t calculated $>$ t tabulated and conclude that the mean is greater than 150.

15. The steps involved in Hypothesis Testing are:

1. State H_0 and H_1
2. State level of significance (usually $\alpha = 0.05$ or 0.01)
3. Decide on the nature of sampling distribution (z or t) and calculate numerical value of TS. Obtain its absolute value.
4. Note the sidedness of the test
5. Use the significance level of the test (α), the sidedness of the test and distribution to obtain the critical value from the table.
6. For one-sided test we use α and for two sided test we use $\alpha/2$.
7. For the t test you need degrees of freedom $n-1$

8. Compare the absolute value of the test statistic (TS) and the critical value obtained from the table.
9. Take decision as follows:
 - a. If TS (calculated by you, a servant) is less than critical value obtained from table (made by expert, master). You accept the null hypothesis (servant is less than master).
 - b. If the absolute value of TS is greater the critical or tabulated value ("servant greater than master"), we reject the null hypothesis.

17. (i) Let W_1 = the event of white ball from 1st bag
 W_2 = the event of white ball from 2nd bag
 B_1 = the event of black ball from 1st bag
 B_2 = the event of black ball from 2nd bag

$$P(\text{both balls are white}) \text{ OR } P(W_1 \cap W_2) = P(W_1) * P(W_2) = \frac{4}{6} * \frac{3}{8} = \frac{12}{48} = \frac{1}{4} = 0.25$$

$$(ii) P(B_1 \cap B_2) = P(B_1) * P(B_2) = \frac{2}{6} * \frac{5}{8} = \frac{10}{48} = \frac{1}{4} = 0.21$$

$$(iii) P(\text{one is white and one is black}) = P(W_1 B_2) + P(B_1 W_2) = \left\{ \frac{4}{6} * \frac{5}{8} \right\} + \left\{ \frac{2}{6} * \frac{3}{8} \right\} = \frac{26}{48} = \frac{13}{24} = 0.54.$$

$$19. P = \frac{8}{100} = 0.08$$

$$q = 1 - 0.08 = 0.92$$

$$P(\text{exactly 5 defective bottles}) = P(X = 5) = {}^6C_5 = (0.08)^5 (0.92)^1 = 6(0.0000033)(0.92) = 0.000018$$