COVENANT UNIVERSITY
NIGERIA

TUTORIAL KIT
OMEGA SEMESTER

PROGRAMME: ECONOMICS

COURSE: ECN 225

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ECN225
MATHEMATICS FOR ECONOMISTS II
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1. A firm faces the production function $Q = 120L + 200K - L^2 - 2K^2$ for positive values of Q. It can buy L at ₦5 a unit and K at ₦8 a unit and has a budget of ₦70. What is the maximum output it can produce?

2. Write out the properties of Definite Integrals with at least two examples each.
3. Evaluate $\int_0^4 \left(\frac{1}{1+x} + 2x\right) dx$

4. Find $\int \left(2e^{2x} + \frac{14x}{7x^2+5}\right) dx$
5. Give examples of Improper Integrals with infinite limits of integration and Improper Integrals with infinite integrands
6. If the marginal cost of a firm is given by the following function of output, C'(Q) = $2e^{0.2Q}$ and the fixed cost is $C_F = 90$. Find the total cost C(Q).
7. Given the MC function of a firm to be $MC = \frac{dc}{dq} = 1.2q^2 - 18q + 100$. Derive the total cost function of the firm.
8. A firm faces the production function $Q = 12K^{0.4}L^{0.4}$ and can buy the inputs K and L at prices per unit of ₦40 and ₦5 respectively. If it has a budget of ₦800 what combination of K and L should it use in order to produce the maximum possible output?

Differentiate the following functions.

9. $y = 4x^2 + 2x^3 - x^4 + 0.1x^5$
10. $y = 20 + 4x - 0.5x^2 + 0.01x^3$.
11. $y = 25 - 0.1x^{-2} + 2x^{0.3}$

12. Explain any **FIVE** rules of differentiation.
13. Find $\int e^{4x+3}\, dx$
14. Evaluate $\int e^{x^3 + x^2 + 1}\, dx$
15. Find $\int (3x^5 + 2x^3 + 1)\, dx$

16. Find the point of inflection if any, of the function $y = x^3 + 3x^2 + 3x$
17. Ascertain whether the function $y = f(x) = x^3 - 6x^2 + 12$ has a point of inflection and determine it.
18. With the method of Lagrange multiplier,

    Minimize $z = x^2 - xy + \frac{3}{2}y^2$

    $Subject\ to; x + 2y = 3$

19. Optimize the function $z = 4x + 6y - x^2 - y^2$

20. A firm faces the production function $Q = 20K^{0.4}L^{0.6}$. It can buy inputs K and L for ₦400 a unit and ₦200 a unit respectively. What combination of L and K should be used to maximize output if its input budget is constrained to ₦6,000?

Solution

1. $MP_L = \frac{dQ}{dL} = 120 - 2L$ $MP_L = \frac{dQ}{dL} = 200 - 4K$

For optimal input combination,

$$\frac{MP_K}{P_K} = \frac{MP_L}{P_L}$$

Therefore, substituting MP_K and MP_L and the given input prices,

$$\frac{120 - 2L}{5} = \frac{200 - 4K}{8}$$

$8(120 - 2L) = 5(200 - 4K)$

$960 - 16L = 1{,}000 - 20K$

$20K = 40 + 16L$

$K = 2 + 0.8L$

Substituting (1) into the budget constraint

$5L + 8K = 70$

gives

$5L + 8(2 + 0.8L) = 70$

$5L + 16 + 6.4L = 70$

$11.4L = 54$

$L = 4.74$ (to 2 dp)

Substituting this result into (1)

$K = 2 + 0.8(4.74) = 5.79$

Therefore maximum output is

$Q = 120L + 200K - L2 - 2K2$

$= 120(4.74) + 200(5.79) - (4.74)2 - 2(5.79)2$

$= 1{,}637.28$

This technique can also be applied to consumer theory, where utility is maximized subject to a budget constraint.

2. Find $\int \left(2e^{2x} + \frac{14x}{7x^2+5}\right) dx$

Integrating separating, $2e^{2x}$ is in the form f'(x) $e^{f(x)}$

Thus, $\int 2e^{2x}\, dx = e^{2x} + C_1$

$\int \frac{14x}{7x^2+5}\, dx$ is in the form $\frac{f'(x)}{f(x)} = \ln(7x^2 + 5) + C_2$

Thus, $\int \left(2e^{2x} + \frac{14x}{7x^2+5}\right) dx = e^{2x} + C_1 + \ln(7x^2 + 5) + C_2$

$= e^{2x} + \ln(7x^2 + 5) + C$

3. Recall TC = VC + FC

Total cost function is the integral of the marginal cost function.

$\int 2e^{0.2Q}\, dq = 2\frac{e^{0.2Q}}{0.2} + \mathrm{C} = 2.\frac{1}{0.2}\ (e^{0.2Q}) + C$

$= 10e^{0.2Q} + C$

Given C_F as 90 implies total cost when Q = 0.

Setting Q= 0 yields $10e^{0.2(0)} + C = 90$

Thus, C = 90-10 = 80
Hence, the total cost function = C(Q) = $10e^{0.2Q} + 80$
4. $MP_L = \frac{dQ}{dL} = 12K^{0.4}L^{-0.4}$ $\quad MP_K = \frac{dQ}{dK} = 8K^{-0.6}L^{0.6}$
$Optimal\ input\ mix\ requires$;
$\frac{MP_K}{P_K} = \frac{MP_L}{P_L}$
$Therefore,$
$$\frac{12K^{0.4}L^{-0.4}}{200} = \frac{8K^{-0.6}L^{0.6}}{400}$$

$Cross\ multiplying\ yields$;
4,800K = 1,600L
3K = L
Substituting this result into the budget constraint
200L + 400K = 6,000
gives
200(3K) + 400K = 6,000
600K + 400K = 6,000
1,000K = 6,000
K = 6
Therefore,
$L = 3K = 18$

5. Total cost function of the firm is the integral of the marginal cost. Thus,
TC = $\int MC\ dq = \int 1.2q^2 - 18q + 100\ dq$
= $(1.2)\frac{q^3}{3} - \frac{18q^2}{2} + 100q + C$
= $0.4q^3 - 9q^2 + 100q + C$
The value of C is dependent on the fixed cost and it is necessarily positive. i.e. C > 0
6. The problem is to maximize the function $Q = 12K^{0.4}L^{0.4}$ subject to the budget constraint
$40K + 5L = 800$ (1)
The theory of the firm tells us that a firm is optimally allocating a fixed budget if the last ₦1 spent on each input adds the same amount to output, i.e. marginal product over price should be equal for all inputs. This optimization condition can be written as;

$\frac{MP_K}{P_K} = \frac{MP_L}{P_L}$ (2)

The marginal products can be determined by partial differentiation:
$MP_K = \frac{dQ}{dK} = 4.8K^{-0.6}L^{0.4}$ (3)
$MP_L = \frac{dQ}{dL} = 4.8K^{0.4}L^{-0.6}$ (4)
Substituting (3) and (4) and the given prices for P_K and P_L into (2)
$$\frac{4.8K^{-0.6}L^{0.4}}{40} = \frac{4.8K^{0.4}L^{-0.6}}{5}$$
Dividing both sides by 4.8 and multiplying by 40 gives;
$K^{-0.6}L^{0.4} = 8K^{0.4}L^{-0.6}$

$Multiplying\ bothe\ sides\ by\ K^{0.6}L^{0.6}\ gives;$

$L = 8K$ (5)

Substituting (5) for L into the budget constraint (1) gives;

40K + 5(8K) = 800

40K + 40K = 800

80K = 800

Thus the optimal value of K is

K = 10

and, from (5), the optimal value of L is

L = 80