COVENANT UNIVERSITY
NIGERIA

TUTORIAL KIT
OMEGA SEMESTER

PROGRAMME: PHYSICS

COURSE: PHY 225
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PHY 225: Mathematical Methods in Physics (I)

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1. Show that the set of vectors \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) are linearly independent. If

\[
\begin{align*}
\mathbf{a} &= \mathbf{b} - 2 \mathbf{c} \\
\mathbf{b} &= \mathbf{a} - \mathbf{c} + \mathbf{d} \\
\mathbf{c} &= 2 \mathbf{b} - 2 \mathbf{d}
\end{align*}
\]

Solution

If the set of vectors \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) are linearly independent, then it must satisfy the condition

\[
\begin{align*}
\mathbf{a} &= \mathbf{b} - 2 \mathbf{c} \\
\mathbf{b} &= \mathbf{a} - \mathbf{c} + \mathbf{d} \\
\mathbf{c} &= 2 \mathbf{b} - 2 \mathbf{d}
\end{align*}
\]

Hence,

\[
\begin{align*}
\mathbf{a} &= \mathbf{b} - 2 \mathbf{c} \\
\mathbf{b} &= \mathbf{a} - \mathbf{c} + \mathbf{d} \\
\mathbf{c} &= 2 \mathbf{b} - 2 \mathbf{d}
\end{align*}
\]

Solving the sets of equations give

\[
\begin{align*}
\mathbf{a} &= 3 \\
\mathbf{b} &= 5 \\
\mathbf{c} &= -2
\end{align*}
\]

Hence, the condition \( \mathbf{a} \cdots \mathbf{b} \cdots \mathbf{c} = 0 \) is satisfied. Therefore vectors \( \mathbf{a}, \mathbf{b}, \mathbf{c} \) are linearly independent.

2. Two particles emitting from a source have displacement at any time. Find the relative displacement of the second particle with respect to the first.

\[
\begin{align*}
\mathbf{d_a} &= 8 \mathbf{i} + 5 \mathbf{j} \\
\mathbf{d_b} &= 4 \mathbf{i} + 10 \mathbf{j}
\end{align*}
\]
3. \( \mathbf{2} \& \mathbf{3} \) What unit vector is perpendicular to both \( \mathbf{2} \) and \( \mathbf{3} \)? Find the angle between these vectors.

Solution

The unit vector is perpendicular to both \( \mathbf{2} \) and \( \mathbf{3} \) if it is the unit vector in the direction of \( \mathbf{2} \times \mathbf{3} \).

\[
\mathbf{2} \times \mathbf{3} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & 1 & 3 \\
1 & 4 & -1
\end{vmatrix} = (1 \cdot (-1) - 3 \cdot 4) \mathbf{i} - (2 \cdot (-1) - 3 \cdot 1) \mathbf{j} + (2 \cdot 4 - 1 \cdot 1) \mathbf{k} = -11 \mathbf{i} + 5 \mathbf{j} + 7 \mathbf{k}
\]

Therefore, the required unit vector is

\[
\frac{\mathbf{2} \times \mathbf{3}}{||\mathbf{2} \times \mathbf{3}||} = \frac{-11 \mathbf{i} + 5 \mathbf{j} + 7 \mathbf{k}}{\sqrt{(-11)^2 + 5^2 + 7^2}} = \frac{-11 \mathbf{i} + 5 \mathbf{j} + 7 \mathbf{k}}{\sqrt{123}}
\]

4. The electric field \( \mathbf{E} \) and magnetic field \( \mathbf{B} \) and velocity \( \mathbf{v} \) of a charged particle \( \mathbf{q} \) are given by

\[
\mathbf{E} = 4 \mathbf{i} + 9 \mathbf{j} + 3 \mathbf{k}, \quad \mathbf{B} = 9 \mathbf{i} + 12 \mathbf{j} + 3 \mathbf{k}, \quad \mathbf{v} = 9 \mathbf{i} + 3 \mathbf{j} + 4 \mathbf{k}
\]

Determine the electrostatic and magnetic force on the charged particle.

5. Show that the line of intersection of the planes \( x+2y+3z=0 \) and \( 3x+2y+z=0 \) is equally inclined to the x and z axes and makes an angle \( \xi \) with the y-axis.

A line is given by \( \mathbf{r} = \mathbf{a} + \lambda \mathbf{b} \), \( \mathbf{r} = \mathbf{c} + \mu \mathbf{d} \). Find the coordinates of the point \( P \) at which the line intersects the plane \( x+2y+3z=6 \).

Solution

The vector normal to the plane is \( \mathbf{n} = \mathbf{b} \times \mathbf{d} \).

\[
\mathbf{n} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
2 & 1 & 3 \\
1 & 4 & -1
\end{vmatrix} = (1 \cdot (-1) - 3 \cdot 4) \mathbf{i} - (2 \cdot (-1) - 3 \cdot 1) \mathbf{j} + (2 \cdot 4 - 1 \cdot 1) \mathbf{k} = -11 \mathbf{i} + 5 \mathbf{j} + 7 \mathbf{k}
\]

This means that the line indeed intersects the plane.
To find the point of intersection, we substitute the above equation into the equation 
\[ x + 2y + 3z = 6, \]
so that
\[ 1 + 4 \cdot 2 + 5 \cdot 3 + 6 \cdot 3 = 6 \]
\[ = -\frac{1}{4} \]
\[ + 4 = 0 \]
\[ + 5 = \frac{3}{4} \]
\[ + 6 = \frac{3}{2} \]

Hence the point of intersection is \((0, 1, 2)\) 6.

A particle of mass 2Kg moving with initial velocity of \((2, 4, 1)\) m/s is acted upon by a 
constant force \((2, -1, 4)\) N. Determine the distance and the velocity after 5s. Find the 
time in which the particle reaches the xy-plane.

7. Show that \(\varphi = 3\) satisfies the partial differential equation
\[ \frac{\partial^2 \varphi}{\partial t^2} - \varphi = 0. \]

8. Find the gradient of the scalar field \(\varphi = 3\)

Solution

9. Find the rate of change with respect to distance \(S\) of \(\varphi = 3\) at the point \((1, 2, -1)\) 
in the direction \(\hat{\mathbf{r}} = \hat{\mathbf{x}} + 2\hat{\mathbf{y}} + 3\hat{\mathbf{z}}\). In which direction is the rate of change of \(\varphi\) greatest at 
this point. What is the value in this direction?

10. Find the divergence of the vector field

Solution
11. Find the laplacian of the scalar field $\varnothing = ?$

12. Find the curl of the function $? = ?$

Solution


Solution

By solving for $\varnothing$ and $\varnothing$, we have

Hence, applying same protocol for

Thus
By substituting \( \varphi \), \( \psi \), \( \chi \) into \( \varphi = \psi \cdot \chi \), then

\[
\begin{align*}
\varphi &= (\bar{\psi} - \bar{\chi}) - (\bar{\chi} + \bar{\psi}) + \varphi \\
\text{Hence,} \\
\varphi &= 1 \varphi + 1 \varphi - 1 \varphi.
\end{align*}
\]

15. Evaluate the integral \( \int \) when \( \varphi \) is the circle in the xy plane defined by \( x^2 + y^2 = 0 \).

16. Given that \( \varphi \) when \( t=2 \) and \( \varphi \) when \( t=3 \). Show that

Solution

\[
\begin{align*}
\varphi &= \frac{1}{2} \left( 3 \varphi \varphi + 3 \varphi \varphi \right) \\
&= \frac{1}{2} \left[ 16 + 4 + 9 + 4 + 1 + 4 \right] \\
&= \frac{1}{2} \left[ 29 - 9 \right] = 10
\end{align*}
\]

17. A vector field \( \varphi \) is given as \( \varphi = 4 \psi - 2 \psi + 2 \psi \), \( V \) is the region bounded by the surface \( x=0, y=0, y=6, \ z=x^2, z=4 \). Evaluate the integral \( \int \).

18. State the Stoke's theorem

Solution

Stoke's theorem is the curl analogue of the divergence theorem. It relates the integral of the curl of a vector field over an open surface \( S \) to the line integral of the vector field around the perimeter \( C \) bounding the surface.

19. Proof mathematically that Gauss law is valid for any Gaussian surface

Solution
Assume a cube. The divergence theorem is

\[ \nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 0 \]

we have

\[ \sum_{i=1}^{3} \frac{\partial F_i}{\partial x_i} = 0 \]

Now the volume integral

\[ \int_V \nabla \cdot \mathbf{F} \, dV = \int_V 0 \, dV \]

\[ = 2 \cdot \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 3 \]

The surface integral is described by the six faces of the cube

\[ \int_{S} \mathbf{F} \cdot d\mathbf{S} = 0 \]
It follows that the volume integral is equal to the surface integral.

20. Compute \(^{\star} \int C \mathbf{F} \cdot d\mathbf{r}\), where \(C\) is the curve of intersection of the plane \(x + y = 2\) and the cylinder \(x^2 + y^2 = 1\), oriented in the counterclockwise direction when viewed from above.