# COVENANT UNIVERSITY

OMEGA SEMESTER TUTORIAL KIT
(VOL. 2)

## PROGRAMME: MATHEMATICS

200 LEVEL

Raising A New Generation Of Leaders

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**LIST OF COURSES**

MAT221: Real Analysis II
MAT222: Mathematics Method
*MAT224: Introduction to Numerical Methods
MAT225: Abstract Algebra

***Not included**

# COVENANT UNIVERSITY

**CANAANLAND, KM 10, IDIROKO ROAD**
**P.M.B 1023, OTA, OGUN STATE, NIGERIA.**

**TITLE OF EXAMINATION:** B.Sc EXAMINATION
**COLLEGE:** SCIENCE AND TECHNOLOGY
**DEPARTMENT:** MATHEMATICS
**SESSION:** 2015/2016 **SEMESTER:** OMEGA
**COURSE CODE:** MAT 221 **CREDIT UNIT:** 3
**COURSE TITLE:** REAL ANALYSIS **II**
**INSTRUCTION:** ANSWER ANY FOUR QUESTION **TIME:** 3 HOURS

1. (a) Give the $\varepsilon - \delta$ definition of a continuous function. (3marks)

(b) Show that $f(x) = \frac{1}{x}$ is uniformly continuous on (0, 1). (7 marks)

(c) Show that if $f(x) = x^2$ then f is continuous at $x = 3$ (7.5marks)

2. (a) Show that if $f'(a)$ exists, then f is continuous at a. (6 marks)

(b) Given $f(x) = 6 - x^2$ , find the derivative of $f'(-3)$ from first principle. (5marks)

(c) Given $g(x) = \begin{cases} \frac{1}{2} & \text{if } x = 0 \text{ or } x = 1 \\ 1 - x & \text{if } 0 < x < 1 \end{cases}$.

Determine the maximum or minimum point of g(x) if they exist. If no, give a condition that will guarantee the maximum and minimum points of g(x). (6.5 marks)

3. (a) Find the value of c where 1< c < 3, that satisfy the equation $f(x) = \sqrt{x-1},\ x \in [1, 3]$ (5marks)

(b) Show that the function $f(x) = x^4 + 3x + 1,\ x \in [-2,-1]$ satisfy the intermediate value theorem. (5 marks)

(c) Let f be monotonic on (a, b) and define

$$\alpha = \inf_{a<x<b} f(x) \text{ and}$$
$$\beta = \text{Sup}_{a<x<b} f(x)$$

If f is nondecreasing, then show that $f(a^+) = \alpha$ and $f(b^-) = \beta$ (7.5marks)

4.(a) What do we mean when f is said to be Riemann integrable on [a, b]? (6marks)

(b) Consider the integral $\int_2^4 (x+1)dx$ .

Let the partition $P_n = 2 + \frac{2k}{n}$, k=0,1,2,$\cdots$

(i)Compute $\bigcup(f, P_n)$ (ii) $L(f, P_n)$ . Using (i) and (ii), show that $\int_2^4 (x+1)dx = 8$ (9.5marks)

5.(a) Suppose that f, g are differentiable on [a, b] with f', g' integrable on [a, b] then prove that $\int_a^b f'(x)g(x)dx = [fg]_a^b - \int_a^b f(x)g'(x)dx$ (8marks)

(b) If $f: R \to R$ is continuous, find F'(x) for each of the following functions;

(i) $F(x) = \int_{x^2}^{1} f(t)dt$ (4.5marks)

(ii) $F(x) = \int_{x^2}^{x^3} f(t)dt$ (5marks)

6. (a) Prove that if f is integrable on [a, b] then
$\int_a^b f(x)dx = \lim_{c \to a}\left(\lim_{d \to b^-} \int_c^d f(x)dx\right)$

(b) Evaluate the following integrals

(i) $\int_1^{\infty} \frac{1+x}{x^3}dx$ (ii) $\int_{\infty}^{0} x^2 e^{-x^3} dx$ (iii) $\int_0^{\pi/2} \frac{\cos x}{(\sin x)^{1/3}}dx$ . (11marks)

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**COURSE TITLE:** REAL ANALYSIS **II**
**COURSE COODINATOR:** Prof. S. A. Iyase
**COURSE LECTURERS:** Prof. S. A. Iyase & Dr. K. S. Eke

**MARKING GUIDE**

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## QUESTION 1

(a) Let X be a non empty set and f a function defined on X. Then f is said to be continuous at point $x_0 \in X$ if given $\varepsilon > 0$ , there exists $\delta > 0$ such that

$$|x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon \ .$$ (5marks)

(b) To show that $f(x) = 1/x$ is continuous, we need to find the value of $\delta$ depending on $\varepsilon$ such that for any $\varepsilon > 0$ we find $\delta > 0$ , $\forall x, x_0 \in X$ we have

$|x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \varepsilon$ . (1mark)

Let $x = 1/\delta$, $x_0 = \dfrac{1+\varepsilon}{\delta}$, $f(x) = \delta$ and $f(x) = \dfrac{\delta}{1+\varepsilon}$ then, (1mark)

$$|x - x_0| = \left|\frac{1}{\delta} - \frac{1+\varepsilon}{\delta}\right| = \left|\frac{1-1-\varepsilon}{\delta}\right| = \left|\frac{-\varepsilon}{\delta}\right| = \frac{\varepsilon}{\delta} < \delta$$

$\varepsilon < \delta^2$ (2marks )

$\delta > \sqrt{\varepsilon}$

$$\left| f(x) - f(x_0) \right| = \left| \delta - \frac{\delta}{1+\varepsilon} \right| = \left| \frac{\delta(1+\varepsilon) - \delta}{1+\varepsilon} \right| = \left| \frac{\delta + \delta\varepsilon - \delta}{1+\varepsilon} \right|$$

$$\frac{\delta\varepsilon}{1+\varepsilon} < \varepsilon$$ (2marks)

$$\frac{\delta}{1+\varepsilon} < 1$$

$$\delta < 1+\varepsilon$$

Thus f is continuous. (1mark)

(c) Given $f(x) = x^2$ , we need to show that f is continuous at point x=3. To do this, we show that for any $\varepsilon > 0$ , we find $\delta > 0$ such that $|x-3| < \delta \Rightarrow |f(x) - f(3)| < \varepsilon$ . (1mark)

$$\left| f(x) - f(3) \right| = \left| x^2 - 3^2 \right| = \left| (x+3)(x-3) \right| \le \delta \left| x+3 \right|$$ (1mark)

$$\left| x \right| = \left| x+3-3 \right| = \left| x-3 \right| + 3 \le \delta + 3.$$ (1mark)

Let $\delta < 1$ then we have

$$\left| x \right| \le 1+3 = 4$$ (1mark))

$$\therefore \left| f(x) - f(3) \right| \le \delta \left| x+3 \right| = \delta \left| 4+3 \right|$$

$$7\delta < \varepsilon$$ (2marks)

$$\delta < \varepsilon/7$$

Let $\delta = \min\{1, \varepsilon/7\}$ (1.5marks)

Thus f is continuous.

Question Two

(a) Let $f'(a)$ exists, then we need to show that f is continuous at a. Using the definition of

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$ (1mark)

We have

$$f(x) = (x-a) \times \frac{f(x) - f(a)}{x-a} + f(a)$$ (1mark)

Taking the limit of both sides as $x \to a$ gives

$$\lim_{x \to a} f(x) = \lim_{x \to a} \left[ (x-a) \times \frac{f(x) - f(a)}{x-a} + f(a) \right]$$

$$= \lim_{x \to a} \left[ (x-a) \times \frac{f(x) - f(a)}{x-a} \right] + \lim_{x \to a} f(a)$$ (3marks)

$$= 0 \times \frac{f(x) - f(a)}{x-a} + f(a)$$

$$= f(a)$$

Thus f is continuous at point a. (1mark)

(b) Given $f(x) = 6 - x^2,$ we need to find the derivative of $f'(-3)$ .

Since $f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ then we have (1mark)

$$f'(-3)=\lim_{x\to -3}\frac{(6-x^2)-(6-(-3)^2)}{x-(-3)}$$

$$=\lim_{x\to -3}\frac{(6-x^2)-(6-9)}{x+3}$$

$$=\lim_{x\to -3}\frac{(6-x^2)+3}{x+3}$$ (4marks)

$$=\lim_{x\to -3}\frac{9-x^2}{x+3}=\lim_{x\to -3}\frac{(3+x)(3-x)}{x+3}$$

$$=\lim_{x\to -3}3-x=3-(-3)=6$$

(c) Given $g(x)=\begin{cases}1/2 & x=0 \text{ or } x=1\\ 1-x & 0<x<1\end{cases}$

$\underset{0<x<1}{Sup}\, g(x)=1$

(2marks)

$\underset{0<x<1}{Inf}\, g(x)=0$

The function has no maximum or minimum points. (1mark)

For the function to have these values we need to alter the condition on $g(x)=1-x,\ 0<x<1$ to $g(x)=1-x,\ 0\le x\le 1$. (2marks)

In this case, $g(x)$ has its maximum point to be 1 and minimum point to be 0.(1.5mks)

Question Three

(a) Given $f(x)=\sqrt{x-1},\ [1,3]$, we need to find the value of c that satisfy the equation.

$f(x)=(x-1)^{1/2}$

$f'(x)=\frac{1}{2}(x-1)^{-1/2}\times 1=\frac{1}{2(x-1)^{1/2}}$ (2marks)

$f'(c)=\frac{1}{2(c-1)^{1/2}}$

But

$f'(c)=\frac{f(b)-f(a)}{b-a}$ (1mark)

This gives

$$f^{'}(c)=\frac{(3-1)^{1/2}-(1-1)^{1/2}}{3-1}$$

$$\frac{1}{2(c-1)^{1/2}}=\frac{2^{1/2}}{2}$$

$$\frac{1}{(c-1)^{1/2}}=2^{1/2}$$

$$1=2^{1/2}(c-1)^{1/2}$$

Take square of both sides.

$$1=2(c-1)$$

$$1=2c-2$$

$$3=2c$$ (2marks)

$$c=3/2$$

Thus the equation is satisfied.

(b) Given $f(x)=x^4+3x+1,\ [-2,-1]$ we need to show that it satisfy the intermediate value theorem.

$$f(x)=x^4+3x+1$$

$$f^{'}(x)=4x^3+3$$

$$f^{'}(c)=4c^3+3$$ (1mark)

$$f^{'}(b)=4b^3+3$$

$$f^{'}(a)=4a^3+3$$

$$f^{'}(-2)=4(-2)^3+3=4\times-8+3=-32+3=-29$$ (2marks)

$$f^{'}(-1)=4(-1)^3+3=-4+3=-1$$

$$f^{'}(c)=\frac{f(b)-f(a)}{b-a}=\frac{(1-3+1)-(16-6+1)}{-1-(-2)}$$

$$=\frac{-3-11}{-1+2}=-14$$ (1mark)

Since $-14\in[-29,-1]$

The IVP Theorem is satisfied. (1mark)

(c) If f is monotonic on (a, b) and define by

$$\alpha=\inf_{a<x<b} f(x)$$

and

$$\beta=\operatorname{Sup}_{a<x<b} f(x)$$

Suppose f is nondecreasing then we need to show that $f(a^+)=\alpha$ and $f(b^-)=\beta$ .

Proof: We show that $f(a^+)=\alpha$ . If $M>\alpha,$ there is $x_0$ in (a, b) such that $f(x_0)<M$ .Since f is nondecreasing, $f(x)<M$ if $a<x<x_0$ . Thus, if $\alpha=-\infty$ then $f(a^+)=\infty$.
If $\alpha>-\infty$ , let $M=\alpha+\varepsilon$ where $\varepsilon>0$ . Then
$\alpha\le f(x)<\alpha+\varepsilon$ , so
$\left|f(x)-\alpha\right|<\varepsilon$ if $a<x<x_0$ . (2marks)

If $\alpha=-\infty$ , this implies that $f(-\infty)=\alpha$. If $\alpha>-\infty$ let $\delta=x_0-a$ . Then the above inequality is equivalent to
$\left|f(x)-\alpha\right|<\varepsilon$ if $a<x<a+\delta$.

Thus $f(a^+)=\alpha$. (2marks)

Next, we show that $f(b^-)=\beta$
If $M<\beta,$ there is $x_0$ in (a, b) such that $f(x_0)>M$ .Since f is nonincreasing, $f(x)>M$ if $x_0<x<b.$ . Thus, if $\beta=\infty$ then $f(b^-)=\infty$.. If $\beta<\infty$ , let $M=\beta-\varepsilon$ where $\varepsilon>0$ . Then
$\beta-\varepsilon\le f(x)<\beta$ , so
$\left|f(x)-\beta\right|<\varepsilon$ if $x_0<x<b$ . (2marks)

If $b=\infty$ , this implies that $f(\infty)=\beta$. If $b<\infty$ let $\delta=b-x_0$ . Then the above inequality is equivalent to
$\left|f(x)-\beta\right|<\varepsilon$ if b-$\delta<x<b$.

Thus $f(b^-)=\beta.$ (1.5marks)

Question Four

(a) F is Riemann integrable on [a, b] if the infimum of upper sums through all partitions of [a, b] is equal to the supremum of all lower sums through all partitions of [a, b]. (6marks)

$$\cup(f,P_n)=\sum_{k=1}^{n} f(x_k)(x_k-x_{k-1})$$

$$f(x_k)=x_k+1=2+\frac{2k}{n}+1$$

$$x_k=2+\frac{2k}{n}$$ (2marks)

(b) (i) $x_{k-1}=2+\dfrac{2k-1}{n}$

$$x_k-x_{k-1}=\frac{2}{n}$$

$$\cup(f,P_n)=\sum_{k=1}^{n}(2+\frac{2k}{n}+1)\frac{2}{n}=\frac{6}{n}\sum_{k=1}^{n}1+\frac{4}{n^2}\sum_{k=1}^{n}k=\frac{6}{n}\times n+\frac{4}{n^2}n(n+1)$$

$$=6+\frac{2(n+1)}{n}$$ (2marks)

$$L(f,P_n)=\sum_{k=1}^{n} f(x_{k-1})(x_k-x_{k-1})$$

(ii) $=\sum_{k=1}^{n}(2+\frac{2(k-1)}{n}+1)\times\frac{2}{n}=\frac{6}{n}\sum_{k=1}^{n}1-\frac{4}{n^2}\sum_{k=1}^{n}1+\frac{4}{n^2}\sum_{k=1}^{n}k$

$=\frac{6}{n}\times n-\frac{4}{n^2}\times n+\frac{4}{n^2}\frac{n(n+1)}{2}=6-\frac{4}{n}+\frac{2(n+1)}{n}$ (2marks)

$\inf_p \bigcup(f,P)\le\lim_{n\to\infty}\{\bigcup(f,P_n)\}\le\lim_{n\to\infty}(6+\frac{2(n+1)}{n})=8$ (2marks)

$\sup_p\{\bigcup(f,P)\}\ge\lim_{n\to\infty}\{L(f,P_n)\}=\lim_{n\to\infty}(6-\frac{4}{n}+\frac{2(n+1)}{n})=8$

Thus $\sup_p\{\bigcup(f,P)\}=\inf_p\{\bigcup(f,P)\}=8$ (1.5marks)

Question Five

(a) Proof: By the product rule

$(f(x)g(x))'=f'(x)g(x)+f(x)g'(x)$ (2marks)

For $x\in[a,b]$. Since f, g are continuous on [a, b] and f', g' are integrable on [a, b] it follows that (fg)' is a sum of integrable functions and hence integrable on [a, b]. Thus by the fundamental theorem of Calculus (2marks)

$$\int_a^b (f(x)g(x))'dx=\int_a^b f'(x)g(x)dx+\int_a^b f(x)g'(x)dx$$

$\left[f(x)g(x)\right]_a^b=\int_a^b f'(x)g(x)dx+\int_a^b f(x)g'(x)dx$ (2marks)

$\int_a^b f'(x)g(x)dx=\left[fg\right]_a^b-\int_a^b f(x)g'(x)dx$ (2marks)

(b) (i)

$$F(x)=\int_{x^2}^{1} f(t)dt$$

$F'(x)=f(1)\frac{d(1)}{dx}-f(x^2)\frac{dx^2}{dx}=-2xf(x^2)$ (4.5marks)

$$F(x)=\int_{x^2}^{x^3} f(t)dt$$

(ii) $F'(x)=f(x^3)\frac{dx^3}{dx}-f(x^2)\frac{dx^2}{dx}$ (3marks)

$=3x^2f(x^3)-2xf(x^2)$ (2marks)

Question Six

(a) $F(x)=\int_a^x f(t)dt$ is continuous on [a, b]. Thus

$$\int_a^b f(x)dx = F(b)-F(a)$$

$$=\lim_{c\to a^+}(\lim_{d\to b^-}(F(d)-F(c)))$$ (3marks)

$$=\lim_{c\to a^+}(\lim_{d\to b^-}\int_c^d f(x)dx)$$ (3.5marks)

(b) (i) $$\int_1^\infty \frac{1+x}{x^3}dx=\lim_{b\to\infty}\int_1^b(\frac{1}{x^3}+\frac{1}{x^2})dx$$

$$=\lim_{b\to\infty}\left[-\frac{1}{2x^2}-\frac{1}{x}\right]_1^b=\lim_{b\to\infty}\left[-\frac{1}{2b^2}-\frac{1}{b}\right]-\left[-\frac{1}{2}-1\right]=\frac{3}{2}$$ (3marks)

(ii) $$\int_\infty^0 x^2e^{x^3}dx=\lim_{a\to\infty}\int_a^0 x^2e^{-x^3}dx$$ (1mark)

Let $u=x^3,\ du=3x^2dx$

If $x=0\Rightarrow u=0,\ \text{ if }\ x=a\Rightarrow u=a^3$ (1mark)

$$\int_\infty^0 x^2e^{x^3}dx=\lim_{a\to\infty}\int_{a^3}^0 x^2e^{-u}\times\frac{du}{3x^2}=\lim_{a\to\infty}\int_{a^3}^0\frac{e^{-u}du}{3}$$ (1mark)

$$=\lim_{a\to\infty}\left[\frac{-e^{-u}}{3}\right]_{a^3}^0=\lim_{a\to\infty}\left[\frac{-1}{3}+\frac{e^{-a^3}}{3}\right]=-\frac{1}{3}$$ (1mark)

(iii) $$\int_0^{\pi/2}\frac{\cos x}{(\sin x)^{1/3}}dx=\lim_{a\to 0}\int_a^{\pi/2}\frac{\cos x}{(\sin x)^{1/3}}dx$$ (1mark)

Let $u=\sin x,\ du=\cos x dx$

If $x=\pi/2,\ u=1,$ If $x=a, u=\sin a$ (1mark)

$$\int_0^{\pi/2}\frac{\cos x}{(\sin x)^{1/3}}dx=\lim_{a\to 0}\int_{\sin a}^1\frac{\cos x}{u^{1/3}}\times\frac{du}{\cos x}=\lim_{a\to 0}\int_{\sin a}^1 u^{-1/3}\text{du}$$ (1mark)

$$=\lim_{a\to 0}\left[\frac{3}{2}u^{2/3}\right]_{\sin a}^1=\lim_{a\to 0}\left[\frac{3}{2}-\frac{3}{2}(\sin a)^{2/3}\right]_{\sin a}^1=\frac{3}{2}$$ (1mark)

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**COLLEGE:** Science and Technology
**SCHOOL:** NATURAL AND APPLIED SCIENCES
**DEPARTMENT:** MATHEMATICS
**SESSION:** 2015/2016 **SEMESTER:** OMEGA
**COURSE CODE:** MAT 222 **CREDIT UNIT:** 3
**COURSE TITLE:** MATHEMATICAL METHODS II
**INSTRUCTION:** Answer Any Four Questions **TIME:** 3 HOURS

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1a. Evaluate $\int\int_R cos(x^2+y^2)dA$ where $R$ is the region that has is above the $x$ axis within the curve $x^2+y^2=9$. (4 marks)

b. Evaluate $\int\int_R x\sin(x+y)dydx$, where $0\le x\le\frac{\pi}{6}$, $0\le y\le x$ (3 marks)

cii. Evaluate $\int\int_R (4x+8y)dA$, where $R$ is the parallelogram with vertices (-1,3), (1,-3), (3,-1) and (1,5). Use the change of variable $x=\frac{1}{4}(u+v)$, $y=\frac{1}{4}(v-3u)$ (10.5 marks)

2a. State and prove Leibnitz's rule for differentiating definite integrals with constant limits. (4 marks)

b. Find the derivative with respect to y of the integral

$I(y)=\int_y^{y^2}\frac{sinyt}{t}dt$ (5 marks)

c. Find the series solution around $x_0=0$ for the following differential equation.

$y''-xy=0$ (8.5 marks)

3a. Let $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$, derive the coefficients

i. $a_0$, ii. $a_n$, and iii. $b_n$. (7 marks)

b. Let $f(x)$ be the function of period 2L given by

$$f(x)=\begin{cases}0 & -2<x<0\\ 2-x & 0<x<2\end{cases}$$

Find the Fourier series (10.5 marks)

4a. Evaluate the following

(i) $(D^2+2D-3)\{e^{3x}\}$ (2 marks)

(ii) $(D+4)\{e^{4x}x^2\}$ (2 marks)

(iii) $\frac{1}{(D^2-3)}\{cos2x\}$ (2 marks)

b. If $I(\alpha)=\int_0^1 \frac{x^{\alpha}-1}{lnx}dx, \alpha>-1$ what is the value of $I(0)$? Show that $\frac{d}{d\alpha}x^{\alpha}=x^{\alpha}lnx,$ and deduce that $\frac{d}{d\alpha}I(\alpha)=\frac{1}{\alpha+1}$ (7.5 marks)

c. Find the derivative with respect to x of the integral

$I(x)=\int_{3-x}^{x^2}(x-t)dt$ (4 marks)

5ai. Define Even and Odd functions. (4 marks)

ii. State two calculus properties of even and odd functions. (4 marks)

iii. State three Algebra properties of even and odd functions. (3 marks)

b. Find the Fourier sine series of the function

$$f(x)=\begin{cases}x & 0\le x<1\\ 1 & 1\le x<2\end{cases}$$ (6.5 marks)

6a. Using the D-Operator method, find the general solution of the following differential equation $y''+3y'+2y=sin2x$ (7 marks)

bi. State the orthogonality conditions of sine and cosine. (4.5 marks)

ii. Evaluate the double integral $\int_0^{\frac{\pi}{2}}\int_0^{\cos\theta} e^{\sin\theta}dr\,d\theta$ (3 marks)

iii. Evaluate the triple integral $\iiint_E yz\cos(x^5)\,dzdydx$ where

$E=\{(x,y,z)\,|\,0\le x\le 1, 0\le y\le x, x\le z\le 2x\}$ (3 marks)

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**SESSION:** 2015/2016 **SEMESTER:** OMEGA
**COURSE CODE:** MAT 225 **CREDIT UNIT:** 3
**COURSE TITLE:** ABSTRACT ALGEBRA
**INSTRUCTION:** ANSWER ONLY FOUR QUESTIONS **TIME:** 3HOURS

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1 (a) Solve the congruence 84X $\equiv$ 24 (mod 180). (5 marks) (b)
Find all integers n for which 13 | $4(n^2 + 1)$. (5 marks) (c)
Show that if $a \mid b$ and $a \mid c$, then $a \mid bx + cy \quad \forall\, x, y \in$ □ (7.5 marks)

2. (a) Given sets S and T such that S=(1,3,9,4,2) and T=(3,7,4.8), write out all the members of the Cartesian product of the sets.
(b) What do you understand by an ISOMORPHISM OF RINGS ? Give an example (5.5marks)
(c) Show that for any integer a, b; b > 0, there exist unique integers q, r such that a = bq +r;
$0 \le r \le b$ (7 marks)

3. (a) Give a comprehensive description of a BOOLEAN ALGEBRA (7 marks)
(b) Prove that there are infinitely many PRIMES (6 marks)
(c) What is the g.c.d of two integers a and b ? (4.5 marks)

4. (a) What do you understand by the following:
i) a Function
ii) set of Rational numbers
iii) set of Integers
iv) set of Prime numbers (4 marks)
(b) Let $R$ be a relation on the set $X$ . When is $R$ said to be an equivalence relation? (3 marks)
(c) Verify whether the relation defined by $R = \{(x, y) \in$ □ $\times$ □ $: x - y$ is a rational number$\}$ is an equivalence relation. (4.5 marks)
(d) Answer the following questions with either Yes or No and give an example for each to explain your answer:

i) Is division a binary operation on the set of real numbers?

ii) Is addition a binary operation on the set of odd numbers?

iii) Is the operation of subtraction a commutative binary operation? (6 marks)

5. (a) Let $(G,*)$ and $(H,*)$ be two groups and let $f: G \to H$ be a function. When is $f$ said to be
   i) a homomorphism (2 marks)
   ii) an isomorphism (2 marks)

   (b) Define the following terms and give two examples of each
   i) semigroup
   ii) monoid
   iii) group
   iv) abelian group (8 marks)

   (c) Show that the set $S = \{1, -1, i, -i\}$ is a group with respect to multiplication operation where $i = \sqrt{-1}$ (5.5 marks)

6. (a) When is a non-empty set, say $R$, said to form a ring? (2.5 marks)

   (b) Generate the addition and multiplication tables for $Z_8$. (2 marks)

   (c)i Define the ideal of a ring. (2 marks)
   ii) Let $(\mathbb{Z}, +, \cdot)$ be a ring. Consider the subset $5\mathbb{Z}$ of $\mathbb{Z}$ defined by $5\mathbb{Z} = \{\cdots, -10, -5, 0, 5, 10, \cdots\}$.
   Show that $5\mathbb{Z}$ is an ideal. (3 marks)

   (d) Define the following terms and give an example of each
   i) zero divisor
   ii) integral domain
   iii) division ring
   iv) field (8 marks)