

# A note on transient convection with chemical reaction and radiative heat transfer past a flat porous plate moving through a binary mixture

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## Abstract

In this article, the problem of transient convection with chemical reaction and radiative heat transfer past a flat porous plate moving through a binary mixture in an optically thin environment is investigated. The dimensionless governing equations for this investigation are solved numerically by the fourth-order Runge–Kutta integration scheme along with shooting technique. Graphical results for velocity, temperature and concentration profiles based on the numerical solutions are presented and discussed.

**Keywords:** Boundary layer flow; Radiative heat transfer; Binary mixture; Arrhenius kinetics; Porous plate; transient convection.

## Nomenclature

$(x, y)$	Cartesian coordinates	$C_\infty$	free stream concentration
$(u, v)$	velocity components	$C$	concentration of the fluid
$T_w$	surface temperature	$C_w$	surface concentration
$T_\infty$	free stream temperature	$D_a$	Damköhler number
$g$	gravitational acceleration	$k$	thermal conductivity
$Q$	heat generation coefficient	$c_p$	specific heat at constant pressure
$T$	fluid temperature	$D$	diffusion coefficient
$R_A$	$n$ th order irreversible reaction	$U_0$	plate uniform velocity
$R_G$	universal gas constant	Pr	Prandtl number
$c$	suction parameter	$E$	activation energy
$S_c$	Schmidt number	$b$	heat generation parameter
$G_r$	thermal Grashof number	$G_c$	solotal Grashof number

## **Greek symbols**

$\theta$	fluid temperature
$\varphi$	fluid concentration
$\eta$	similarity variable
$\gamma$	activation energy parameter
$\beta$	thermal volumetric-expansion coefficient
$\beta_c$	concentration volumetric-expansion coefficient
$\sigma$	Stefan-Boltzmann constant
$\alpha$	absorption coefficient
$\rho$	fluid density
$\nu$	kinematic viscosity

## **1. Introduction**

Studies related to boundary layer flow of a binary mixture of fluids are always important in view of its applications in various branches of engineering and technology. A familiar example is an emulsion which is the dispersion of one fluid within another fluid. Typical emulsions are oil dispersed within water or water within oil. Another example where the mixture of fluids plays an important role is in multigrade oils. Polymeric type fluids are added to the base oil so as to enhance the lubrication properties of mineral oil [1]. Several problems relating to the mechanics of oil and water emulsions, particularly with regard to applications in lubrication practice, have been considered within the context of the binary mixture theory by Al- Sharif et al. [2] and Wang et al. [3].

Meanwhile, analyses of the transport processes and their interaction with chemical reactions and thermal radiation are quite difficult and are intimately connected to the underlying fluid dynamics. Such a combined analysis of chemical and physical processes constitutes the core of chemical reaction engineering. Moreover, the study of heat generation or absorption effects in moving fluids is important in view of several physical

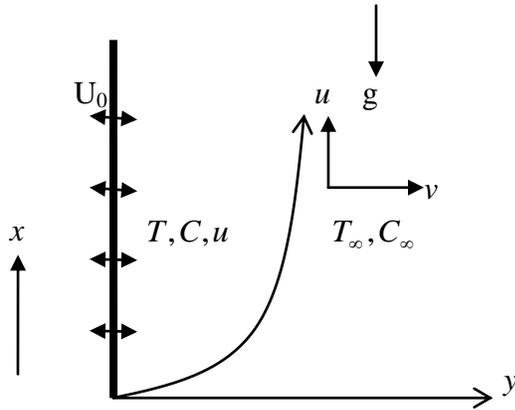
problems, such as fluid undergoing exothermic or endothermic chemical reaction. The recent advances in understanding physics of flows and computational flow modeling can make tremendous contributions in engineering and industrial processes. Sparrow and Cess [4] studied the problem of absorbing – emitting gray fluid with a black vertical plate. Using perturbation technique, they obtained solution that was applicable for small values of the conduction-radiation interaction parameter. Abdus Sattar and Hamid Kalim [5] investigated the unsteady free convection interaction with thermal radiation in a boundary layer flow past a vertical porous plate. They found that an increase in Prandtl number might lead to a reduction in the thermal boundary layer thickness. Makinde [6] studied the transient free convection interaction with thermal radiation of an absorbing-emitting fluid along a moving vertical permeable plate. The problem of unsteady hydromagnetic convection through a porous medium with combined heat and mass transfer with heat source / sink was investigated by Kamel [7]. Kandasamy et al. [8] studied the combined effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection. Their result shows that the flow field is influenced appreciably by chemical reaction, heat source and suction or injection at the wall of the wedge.

In this present work, we investigate the transient convection with chemical reaction and radiative heat transfer past a flat porous plate moving through a binary mixture which is an extension of Kandasamy et al. [8] to a binary mixture instead of at the wall of the wedge. The similarity transformation has been utilized to convert the governing partial differential equations into ordinary differential equations and then the numerical solution of the problem is drawn using the fourth-order Runge–Kutta integration scheme along

with shooting method. The analysis of the results obtained shows that the flow field is influenced appreciably by the presence of chemical reaction, heat source, thermal radiation at the plate surface. It is hoped that the results obtained will provide useful information for applications.

## 2. Mathematical formulation

Consider the unsteady one – dimensional convective flow with chemical reaction and radiative heat transfer past a vertical porous plate moving through a binary mixture (Fig. 1). The flow is assumed to be in the  $x$ -direction, which is taken along the vertical plate.



**Fig. 1:** Flow configuration and coordinate system.

We choose Cartesian axes  $(x, y)$  parallel and perpendicular to the plate, respectively and velocity components are  $(u, v)$ , respectively. Then, the governing equations for such a flow are [9-12],

$$\frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta_c(C - C_\infty), \quad (2)$$

$$\rho C_p \left( \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + Q + 4\sigma\alpha T^4, \quad (3)$$

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + R_A, \quad (4)$$

where  $u$  and  $v$  are velocity components in  $x$  and  $y$  directions respectively,  $T$  is the temperature,  $t$  is the time,  $g$  is the acceleration due to gravity,  $\beta$  is the thermal expansion coefficient,  $\beta_c$  is the concentration expansion coefficient,  $\nu$  is the kinematic viscosity,  $D$  is the chemical molecular diffusivity,  $k$  is the thermal conductivity,  $\rho$  is the density,  $T_w$  is the wall temperature,  $T_\infty$  is the free stream temperature,  $C_w$  is the species concentration at the plate surface,  $C_\infty$  is the free stream concentration,  $Q = (-\Delta H)R_A$  is the heat of chemical reaction and  $\Delta H$  is the activation enthalpy. We employed Arrhenius type of the  $n^{\text{th}}$  order irreversible reaction given by,

$$R_A = k_r e^{-E/R_G T} C^n, \quad (5)$$

where  $k_r$  is the chemical reaction rate,  $R_G$  is the universal gas constant and  $E$  is the activation energy parameter. The appropriate initial and boundary conditions are

$$u(y,0) = 0, \quad T(y,0) = T_w, \quad C(y,0) = C_w, \quad (6)$$

$$u(0,t) = U_0, \quad T(0,t) = T_w, \quad C(0,t) = C_w, \quad t > 0, \quad (7)$$

$$u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \quad \text{as } y \rightarrow \infty, t > 0, \quad (8)$$

where  $U_0$  is the plate characteristic velocity. We introduce the following dimensionless quantities and parameters,

$$\begin{aligned}
u &= U_0 F(\eta), \quad (\theta, \theta_w) = \frac{(T, T_w)}{T_\infty}, \quad (\varphi, \varphi_w) = \frac{(C, C_w)}{C_\infty}, \quad G_r = \frac{4vtg\beta T_\infty}{U_0}, \\
G_c &= \frac{4vtg\beta_c C_\infty}{U_0}, \quad P_r = \frac{\nu}{\lambda}, \quad \lambda = \frac{k}{\rho c_p}, \quad S_c = \frac{\nu}{D}, \quad \gamma = \frac{E}{R_G T_\infty}, \quad \eta = \frac{y}{2\sqrt{vt}}, \\
k_0 &= k_r e^{-\frac{E}{R_G T_\infty}}, \quad b = \frac{(-\Delta H)C_\infty}{\rho c_p T_\infty}, \quad Ra = \frac{16\sigma\alpha t T_\infty^3}{\rho c_p}, \quad Da = 4tk_0 C_\infty^{n-1}.
\end{aligned} \tag{9}$$

From Eq. (1),  $v$  is either constant or a function of time. Following Makinde [6], we choose

$$v = -c \left( \frac{\nu}{t} \right)^{\frac{1}{2}}, \tag{10}$$

where  $c > 0$  is the suction parameter and  $c < 0$  is the injection parameter. Eqs. (2) – (4) then become

$$F'' + 2(\eta + c)F' = -G_r(\theta - 1) - G_c(\varphi - 1), \tag{11}$$

$$\frac{1}{P_r}\theta'' + 2(\eta + c)\theta' = bD_a\varphi^n \exp\left(\gamma\left(1 - \frac{1}{\theta}\right)\right) - Ra\theta^4, \tag{12}$$

$$\frac{1}{S_c}\varphi'' + 2(\eta + c)\varphi' = -D_a\varphi^n \exp\left(\gamma\left(1 - \frac{1}{\theta}\right)\right), \tag{13}$$

with the boundary conditions

$$\begin{aligned}
F(0) &= 1, \quad \theta(0) = \theta_w, \quad \varphi(0) = \varphi_w, \\
F(\infty) &= 0, \quad \theta(\infty) = 1, \quad \varphi(\infty) = 1,
\end{aligned} \tag{14}$$

where prime symbol represents derivatives with respect to  $\eta$ ,  $b$  is the heat generation parameter,  $Da$  is the Damköhler number,  $Ra$  is the radiation parameter,  $\gamma$  is the activation energy parameter,  $G_r$  is the thermal Grashof number and  $G_c$  solutal Grashof number.

### 3. Computational approach

The set of non-linear ordinary differential equations (11)–(13) with boundary conditions in (14) have been solved numerically by using the Runge–Kutta integration scheme with a modified version of the Newton–Raphson shooting method with  $\gamma$ ,  $Ra$ ,  $Da$ ,  $\theta_w$ ,  $\varphi_w$ ,  $b$ ,  $n$ ,  $Sc$ , and  $Pr$  as prescribed parameters. The computations were done by a program which uses a symbolic and computational computer language MAPLE [9]. A step size of  $\Delta\eta = 0.001$  was selected to be satisfactory for a convergence criterion of  $10^{-7}$  in nearly all cases. The value of  $y_\infty$  was found to each iteration loop by the assignment statement  $\eta_\infty = \eta_\infty + \Delta\eta$ .

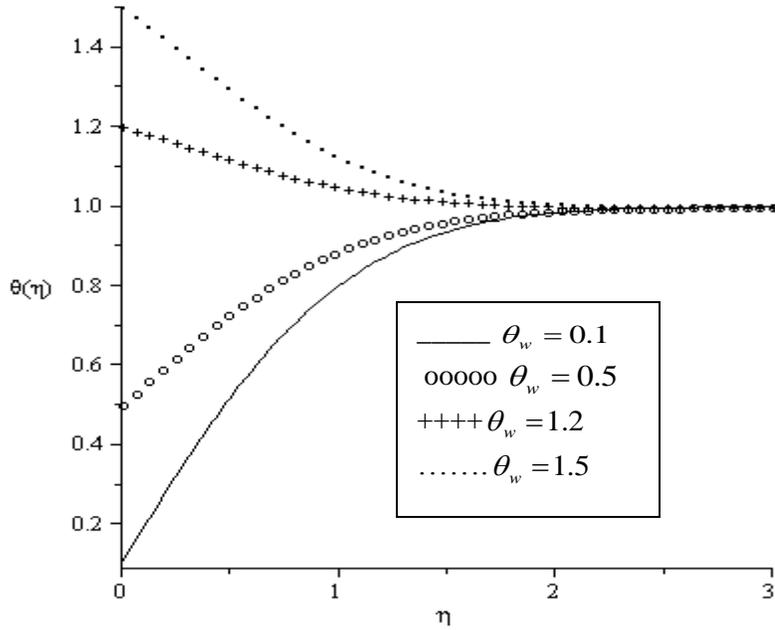
### 4. Analysis of results and conclusion

In order to get a clear insight of the physical problem, the velocity, temperature and concentration have been discussed by assigning numerical values to the parameters encountered in the problem. To be realistic, the values of Schmidt number ( $Sc$ ) are chosen for hydrogen ( $Sc = 0.22$ ), water vapour ( $Sc = 0.62$ ), ammonia ( $Sc = 0.78$ ) and Propyl Benzene ( $Sc = 2.62$ ) at temperature  $25^\circ\text{C}$  and one atmospheric pressure. The values of Prandtl number is chosen to be  $Pr = 0.71$  which represents air at temperature  $25^\circ\text{C}$  and one atmospheric pressure. It should be mentioned here that  $Da > 0$  indicates an increase in the chemical reaction rate.

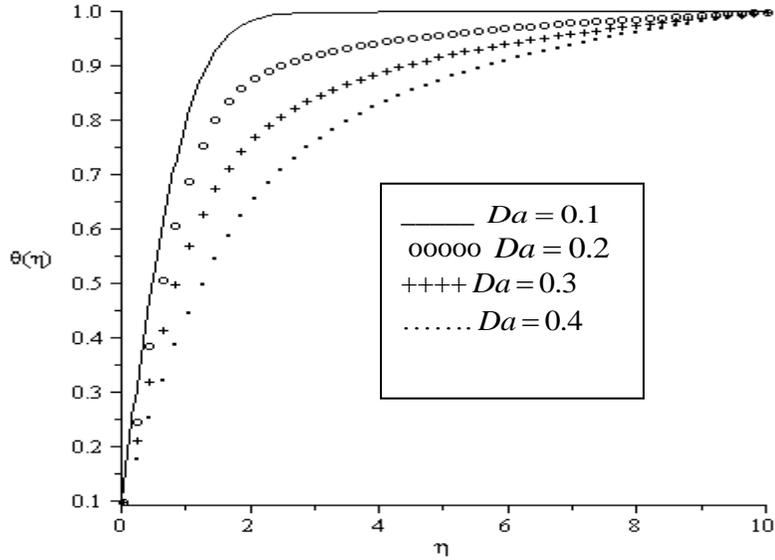
## 4.1 Results discussion

Fig. 2 depicts the variation of temperature profile against spanwise coordinate  $\eta$  for varying values of wall temperature parameter  $\theta_w$  and fixed values of other physical parameter in the presence of uniform suction. It is interesting to note from this figure that the fluid temperature increases (or decreases) toward the free stream temperature whenever the plate temperature is lower (or higher) than the free stream temperature. Fig. 3 shows the effects of chemical reaction rate on the fluid temperature profile within the boundary layer when the wall temperature is lower than the free stream temperature in the presence of uniform suction. A decrease in the fluid temperature is observed with increasing value of  $Da$ . Fig. 4 represents graph of temperature distribution with spanwise coordinate  $\eta$  for different values of radiation parameter  $Ra$ . From this figure, it is seen that the fluid temperature starts from a minimum value at the moving plate surface, increases till it attains the peak value within the boundary layer, and then starts to decrease until it reaches the free stream temperature value at the end of the boundary layer for all the values of radiation parameter. The effect of increasing value of radiation parameter is more prominent at the peak value within the boundary layer i.e. the peak value drastically increases with increasing value in the value of  $Ra$  due to internal heat generation by Arrhenius kinetics. Fig. 5 represents the temperature profiles for different values of reaction order parameter ( $n$ ). The fluid temperature is lowest at the moving plate surface and increases toward the free stream value away from the plate satisfying the boundary condition. It is observed that the fluid temperature decreases with increasing order of Arrhenius chemical reaction.

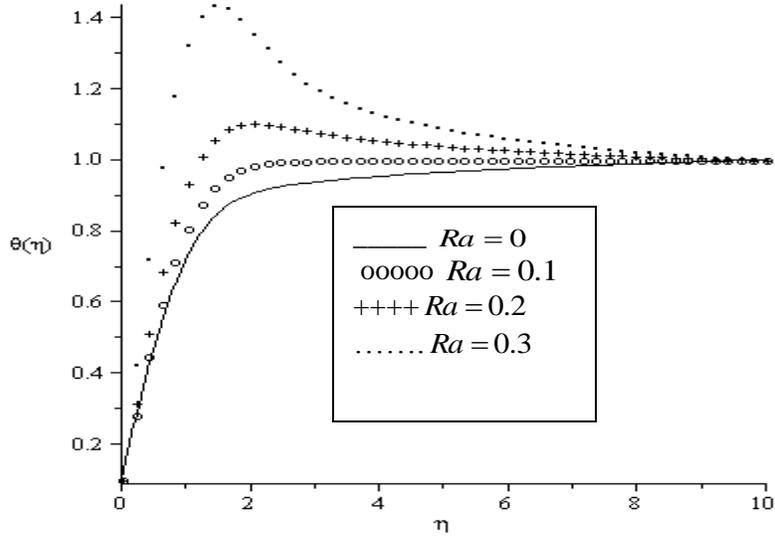
The effect of chemical reaction rate parameter  $Da$  as shown in Fig.6 is very important in concentration field. Chemical reaction increases the rate of interfacial mass transfer. The reaction reduces the local concentration, thus increasing its concentration gradient and its flux. As seen from the graph, increasing parameter values of the chemical reaction rate causes an increase in the concentration of the chemical species in the boundary layer. Moreover, the peak concentration value is attained within the boundary layer and increases with increasing value in the value of  $Da$  due to Arrhenius kinetics. Similar facts are seen in the case when Schmidt number is increased as noted in Fig. 7. We observed from this figure that an increase in the Schmidt number  $Sc$  lead to an increase in the chemical species concentration within the boundary layer. Physically, the increase of  $Sc$  means decrease of molecular diffusion  $D$ . Hence, the concentration of the species is higher for higher values of  $Sc$  and lower for small values of  $Sc$ . Fig. 8 is a plot of concentration profiles against  $\eta$  for various values of the order of reaction  $n$ . As seen from this plot, the concentration of species decreases with increasing value of  $n$  in the solutal boundary layer.



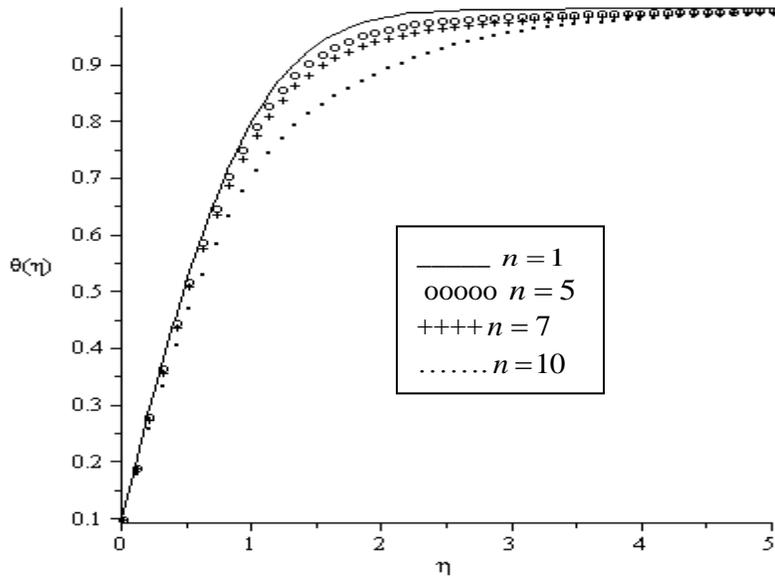
**Fig. 2.** Variation of the boundary layer temperature profiles with increasing values of the wall temperature when  $G_r = G_c = Ra = \gamma = Da = \phi_w = c = 0.1$ ,  $b = n = 1$ ,  $Sc = 0.62$ .



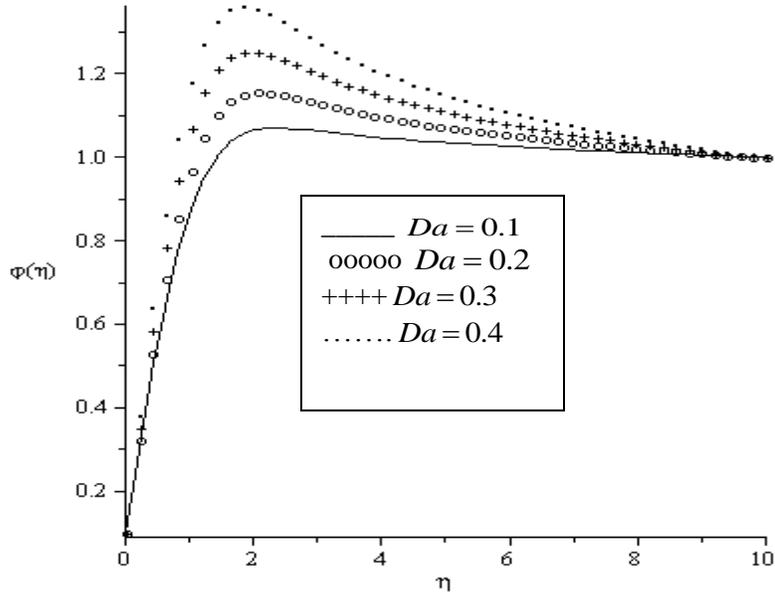
**Fig. 3.** Variation of the boundary layer temperature profiles with increasing values of the Damköhler number when  $G_r = G_c = \gamma = Ra = \theta_w = \phi_w = c = 0.1$ ,  $b = n = 1$ ,  $Sc = 0.62$ .



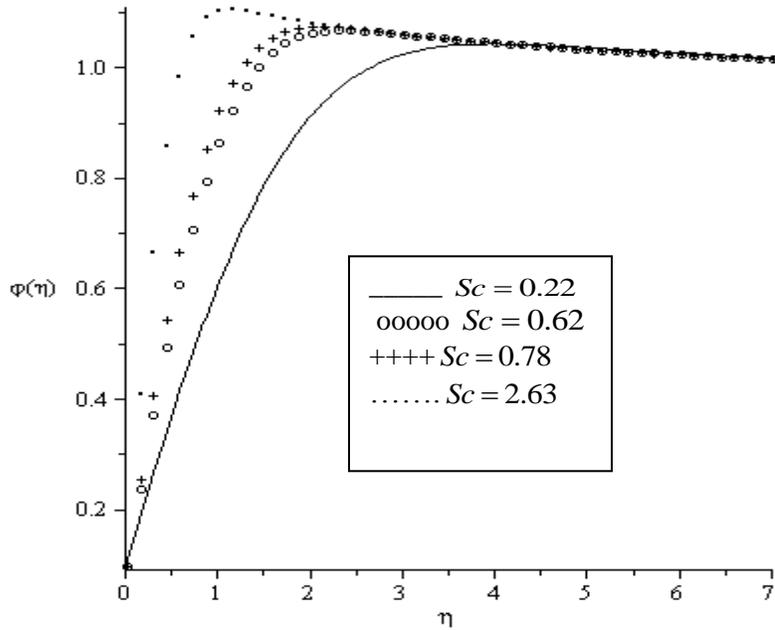
**Fig. 4.** Variation of the boundary layer temperature profiles with increasing values of the Damköhler number when  $G_r = G_c = \gamma = Da = \theta_w = \varphi_w = c = 0.1$ ,  $b = n = 1$ ,  $Sc = 0.62$ .



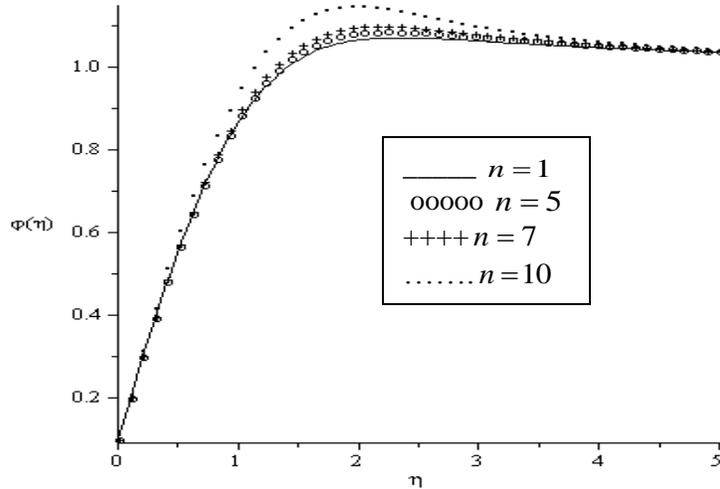
**Fig. 5.** Variation of the boundary layer temperature profiles with increasing values of the reaction order when  $G_r = G_c = Ra = \gamma = Da = \theta_w = \varphi_w = c = 0.1$ ,  $b = 1$ ,  $Sc = 0.62$ .



**Fig. 6.** Variation of the boundary layer concentration profiles with increasing values of the Damköhler number when  $G_r = G_c = \gamma = Ra = \theta_w = \varphi_w = c = 0.1$ ,  $b = n = 1$ ,  $Sc = 0.62$ .



**Fig. 7.** Variation of the boundary layer concentration profiles with increasing values of the Schmidt number when  $G_r = G_c = Ra = \gamma = Da = \theta_w = \varphi_w = c = 0.1$ ,  $b = n = 1$ .



**Fig. 8.** Variation of the boundary layer concentration profiles with increasing values of the reaction order when  $G_r = G_c = Ra = \gamma = Da = \theta_w = \varphi_w = c = 0.1$ ,  $b = 1$ ,  $Sc = 0.62$ .

## 5. Conclusions

The effects of  $n^{\text{th}}$  order Arrhenius chemical reaction, thermal radiation on transient convection of a viscous incompressible fluid past a vertical porous plate is studied. A set of non-linear coupled differential equations governing the fluid velocity, temperature and chemical species concentration is solved numerically for various material parameters. The temperature profile increases in the presence of  $Ra$  and decreases with increasing values of  $Da$  and  $n$ . In addition, the chemical species concentration within the boundary layer increases with increase values of  $Da$ ,  $n$  and  $Sc$ .

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