Synchronous Generator Excitation Chatter-Free Sliding Mode Controller

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Abstract—A chatter-free sliding mode controller (SMC) for synchronous generator excitation is presented in this paper. A linearized model of a single machine connected to an infinite bus is employed to design a variable-structure controller which not only stabilizes the system, but also ensures that this is maintained in the face of system parameter variations. Validating the robustness feature of an SMC, simulation results that show the dynamic performance of the system under both constant excitation and SMC-controlled excitation are presented.

Index Terms—Excitation system control, robustness, sliding model control, synchronous generator model.

I. INTRODUCTION

Electric power systems have witnessed substantial growth—a more transmission networks have been built and greater interconnections among generation areas effected, with new measurement and monitoring devices (occasioned by advancements in technology) being added in various parts of the systems. This growth has resulted in high dimensionality and complexity of the power systems, thereby sparking off intense efforts to make them operate satisfactorily. Key to the operation of an electric power network is stability. And this stability principally is dependent on the ability of synchronous generators in the system to maintain synchronism with one another during both normal operating conditions and abnormal conditions. Principally, synchronism is a function of how the power system reacts to disturbances, i.e., whether it has sufficient restoring forces to balance the disturbing forces [1].

Synchronous generator excitation control design has gone through a range of techniques and strategies, such as linear lead/lag control [2], optimal control [3], [4], feedback linearization control [5], [6], [7], adaptive control [8], [9], [10], robust control [5], fuzzy logic control [11], [12], etc., all of which aimed at damping out oscillations and maintaining both small-signal and large-signal stability of power systems.

This paper examines the performance of a single-machine-infinite-bus system under sliding mode excitation control action when the system is assumed to have both constant and variable parameters. Sliding mode control is known to provide robustness in face of parameter variations through a variable-structure control action which drives the state of a dynamic system towards a surface, called the sliding surface, and causes the state, as it slides along to an equilibrium point, to remain on it [13], [14]. Although implementation of the SMC switching control is usually imperfect and the value of the switching surface is not known with infinite precision, thereby resulting in control signal chattering, several methods are available for providing smooth switching, such as the replacement of the switching function with a saturation function [15], the replacement of a sliding surface with a sliding sector [16], and the use of an asymptotic observer [17]. The saturation function is used to eliminate chattering in this paper.

The rest of the paper is organized as follows. Section II presents the system modeling, stating the power system model used; Section III discusses the design of the switching surface and the control law. Results and discussion of simulation study are presented in Section IV, and conclusions drawn in Section V.

II. SYSTEM MODELING

A single-machine-connected to an infinite-bus system is used in this paper. The machine is represented by a third-order nonlinear model [18]:

\[
\begin{align*}
\dot{\delta} &= \omega \\
\dot{\omega} &= B_1 - A_1\omega - A_2\psi_f\sin\delta - \frac{B_2}{2}\sin2\delta \\
\psi_f &= u - C_1\psi_f + C_2\cos\delta
\end{align*}
\]  

(1)

where \(\delta\) is the rotor angle, \(\omega\) is the speed variation, and \(\psi_f\) is the field flux linkage. Values of system parameters as well as expressions for \(A_1, A_2, B_1, B_2, C_1,\) and \(C_2\) are given in Appendix A. Linearizing the system about the steady-state values [19]

\[\delta_1 = 0.7438, \omega_1 = 0, \psi_{f1} = 7.7438\]

yields
\[
\begin{bmatrix}
\Delta \dot{\delta} \\
\Delta \dot{\omega} \\
\Delta \dot{\psi}_f
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
-64.534 & -0.270 & -8.1533 \\
-1.2896 & 0 & -0.323
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta \omega \\
\Delta \psi_f
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\Delta u
\] (2)

III. SWITCHING SURFACE AND CONTROL LAW DESIGN

The design problem here is to determine the switching surface, s, and the gain matrix \( \mathbf{K} \) such that the system trajectory, from any initial position, is forced to hit the switching line \( s = 0 \) and stay on it as the states slide towards the equilibrium point.

Equation (2) can be written in a form suitable for switching surface and control law design as:

\[
\begin{bmatrix}
\dot{\Omega}_1 \\
\dot{\Omega}_2
\end{bmatrix} =
\begin{bmatrix}
\mathbf{A}_{11} & \mathbf{A}_{12} \\
\mathbf{A}_{21} & \mathbf{A}_{22}
\end{bmatrix}
\begin{bmatrix}
\Omega_1 \\
\Omega_2
\end{bmatrix} +
\begin{bmatrix}
\mathbf{B}_1 \\
\mathbf{B}_2
\end{bmatrix} \mathbf{u}_e
\] (3)

where

\[
\Omega_1 = \begin{bmatrix} \Delta \delta \\ \Delta \omega \end{bmatrix}, \quad \Omega_2 = \begin{bmatrix} \psi_f \end{bmatrix}
\]

\[
\mathbf{A}_{11} = \begin{bmatrix} 0 & 1 \\ -64.534 & -0.270 \end{bmatrix}, \quad \mathbf{A}_{12} = \begin{bmatrix} 0 \\ -8.1533 \end{bmatrix}
\]

\[
\mathbf{A}_{21} = \begin{bmatrix} -2.2896 & 0 \end{bmatrix}, \quad \mathbf{A}_{22} = \begin{bmatrix} -0.323 \end{bmatrix}
\]

\[
\mathbf{B}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \mathbf{B}_2 = \begin{bmatrix} 1 \end{bmatrix}
\]

\[
\mathbf{u}_e = \Delta \mathbf{u}
\]

Given the switching surface

\[
s = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 \end{bmatrix} \begin{bmatrix} \Omega_1 \\ \Omega_2 \end{bmatrix} = 0
\] (4)

and the control law

\[
\mathbf{u}_e = \begin{bmatrix} \mathbf{K}_1 & \mathbf{K}_2 \end{bmatrix} \begin{bmatrix} \Omega_1 \\ \Omega_2 \end{bmatrix}
\] (5)

where \( \mathbf{C}_1 \) and \( \mathbf{K}_1 \) are 1 x 2 matrices, and \( \mathbf{C}_2 \) and \( \mathbf{K}_2 \) are 1 x 1 matrices.

Substitution of (4) into (3) results in the following equation for \( \dot{\Omega}_1 \):

\[
\dot{\Omega}_1 = \left( \mathbf{A}_{11} - \mathbf{A}_{12} \mathbf{C}_2^{-1} \mathbf{C}_1 \right) \Omega_1
\] (6)

This equation carries the property of a feedback-structure system in which \( \mathbf{A}_{11} \) is the state matrix, \( \mathbf{A}_{12} \) the input matrix, and \( \mathbf{C}_2^{-1} \mathbf{C}_1 \) the gain matrix.

Therefore, a pole-placement design technique can be applied to this reduced equation in order to obtain a suitable value for \( \mathbf{C}_2^{-1} \mathbf{C}_1 \), and therefore, \( \mathbf{C}_1 \) when it is assumed that \( \mathbf{C}_2 \) is an identity matrix [8].

Therefore, the switching surface could be computed as

\[
s = -8.0294x_1 - 2.7878x_2 + x_3
\] (7)

where \( [x_1 \ x_2 \ x_3] = [\Delta \delta \ \Delta \omega \ \Delta \psi_f] \).

To design the control law, a Lyapunov function [5], [13], [14]

\[
V = \frac{1}{2} s^2
\] (8)

is chosen such that (9) (given below) is satisfied. This ensures that the system state trajectory reaches and remains on the switching surface.

\[
\dot{V} = ss < 0
\] (9)

By substituting for \( s \) and combining eqns. (4) and (6), the following condition can be written:

\[
[(\mathbf{C}_1 \mathbf{A}_{11} + \mathbf{C}_2 \mathbf{A}_{21}) \Omega_1 + (\mathbf{C}_1 \mathbf{A}_{12} + \mathbf{C}_2 \mathbf{A}_{22}) \Omega_2]
\]

\[
+ [\mathbf{C}_2 \mathbf{B}_2 \mathbf{u}_e] s < 0
\]

Let the control law which satisfies this condition be given as

\[
\mathbf{u}_e = \mathbf{K} \Omega
\] (11)

where

\[
\mathbf{K} = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}
\]

An equivalent control law, \( \mathbf{u}_{\mathbf{eq}} \), can be computed from (3) as

\[
\mathbf{u}_{\mathbf{eq}} = -(\mathbf{C}_2 \mathbf{B}_2)^{-1} [(\mathbf{C}_1 \mathbf{A}_{11} + \mathbf{C}_2 \mathbf{A}_{21}) \Omega_1]
\]

\[
+ [(\mathbf{C}_1 \mathbf{A}_{12} + \mathbf{C}_2 \mathbf{A}_{22}) \Omega_2] = \mathbf{K}_{\mathbf{eq}} \Omega
\] (12)

Therefore, by combining (11) and (12), (10) can be reduced to

\[
\mathbf{C}_2 \mathbf{B}_2 \left( \left( \mathbf{K} - \mathbf{K}_{\mathbf{eq}} \right) \Omega \right) s < 0
\] (13)

From this last expression, the control gains are selected so that

\[
\mathbf{K} \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}
\]

\[
\begin{cases}
< k_{\mathbf{eq}} j \Omega, s > 0 \\
> k_{\mathbf{eq}} j \Omega, s < 0
\end{cases}
\] (14)

By introducing a signum function, the general control law can be formed as:

\[
\mathbf{u}_e = [k_{\mathbf{eq}} 1 - p_1 \text{sgn}(x_1 s)] x_1
\]

\[
+ [k_{\mathbf{eq}} 2 - p_2 \text{sgn}(x_2 s)] x_2
\]

\[
+ [k_{\mathbf{eq}} 3 - p_3 \text{sgn}(x_3 s)] x_3
\] (15)

By solving (12) completely, \( \mathbf{K}_{\mathbf{eq}} \) could be found as

\[
\mathbf{K}_{\mathbf{eq}} = [-177.6183 \ 7.2767 \ -22.4068]
\]
Since $p_1$, $p_2$, and $p_3$ are usually chosen to be positive and large [19], their values are given here to be about twice the absolute values of equivalent control gains.

The control law is now given as:

$$u_e = \begin{bmatrix} -177.6183 - 266.4275\text{sgn}(x_1) \\ 7.2767 - 10.9151\text{sgn}(x_2) \\ -22.4068 - 33.6102\text{sgn}(x_3) \end{bmatrix} x_1 + \begin{bmatrix} x_2 \\ 0 \\ 0 \end{bmatrix} \quad (16)$$

To reduce the level of signal chattering, $\text{sgn}(x_i,s)$ is replaced with $\text{sat}\{(x_i,s)/\varepsilon\}$, where $i = 1, 2, 3$; $\text{sat}\{(x_i,s)/\varepsilon\} = \text{sgn}(x_i,s)$ if $\text{abs}(x_i,s) > \varepsilon$, and $\text{sat}\{(x_i,s)/\varepsilon\} = x_i,s$ if $\text{abs}(x_i,s) < \varepsilon$. And $\varepsilon = 0.7$ is used for the simulation.

IV. SIMULATION RESULTS AND DISCUSSION

The performance of the synchronous generator, when connected to an infinite bus, is verified through simulation under the control action of the chatter-free sliding mode controller. Two simulation scenarios are considered—when all the system parameters are assumed to be fixed, and also, when they are arbitrarily varied. Fig. 1 shows the result of the system response for the first case, whereas Figs. 2 to 4 show the results for the second case. In the later situation, parameters $A_2$, $B_2$, and $C_2$ are varied independently in turn. These graphs reflect the robustness feature of the sliding mode control, as the system responses observed for the various scenarios bear only slight differences. The results also show that the system oscillations are well damped out and that the controller performs well on the non-linear system—very small steady-state error, and restoration of system to the steady-state within 0.5s.

![Fig. 1. Nonlinear system response under SMC-controlled excitation](image-url)
Fig. 2. Nonlinear system behavior under SMC-controlled excitation for 29% reduction in A2.

Fig. 3. Nonlinear system behavior under SMC-controlled excitation for 42% reduction in B2.
A linearized model of third-order nonlinear synchronous generator equations has been employed in this paper to design an SMC-based control system for the excitation of a synchronous generator connected to an infinite bus.

With the aid of an appropriate saturation function, the signal chattering in the system is eliminated. The designed controller guarantees system stability and consistent performance even in the face of parameter uncertainty, although the initial control effort, which can be reduced significantly with proper selection of $p_1$, $p_2$, and $p_3$ in (15), is a bit high.

**APPENDIX A**

The system parameters in pu are given as [13]:

- $X_d = 1.25$  
- $X'_d = 0.3$
- $X_q = 0.7$  
- $T'_{do} = 9.0$
- $M = 0.0185$  
- $D = 0.005$
- $X_e = 0.2$  
- $E = 1.0$

Expressions for parameters $A_1$, $A_2$, $B_1$, $B_2$, $C_1$, and $C_2$ in the system model are [12]:

- $A_1 = \frac{D}{M}$  
- $A_2 = \frac{E}{M(X'_d + X_e)T'_{do}}$
- $B_1 = \frac{P_m}{M}$  
- $B_2 = \frac{E^2(X'_d - X_q)}{M(X'_d + X_e)(X'_d + X_e)}$
- $C_1 = \frac{(X'_d + X_e)}{(X'_d + X_e)T'_{do}}$  
- $C_2 = \frac{(X'_d - X_q)}{(X'_d + X_e)}E$

where

- $D =$ damping coefficient
- $M =$ inertial constant
- $P_m =$ mechanical power input
- $E =$ infinite bus bar voltage
- $X_e =$ transmission line impedance
- $X'_d =$ d-axis synchronous reactance
- $X'_q =$ q-axis synchronous reactance
- $X'_t =$ d-axis transient reactance
- $T'_{do} =$ d-axis transient open-circuit time constant

**V. CONCLUSIONS**

Fig. 4. Nonlinear system behavior under SMC-controlled excitation for 29% reduction in $C_2$. 

![Graphs](image-url)
APPENDIX B

function out1=satfun(y,a)
   if dm==1
      if abs(y)<a
         out1=y;
      else
         out1=sign(y);
      end
   else
      out1=zeros(dm,1);
      for j=1:dm
         if abs(y(j))<a
            out1(j)=y(j);
         else
            out1(j)=sign(y(j));
         end
      end
   end

REFERENCES


Ayoade F. Aghetuyi obtained a Bachelor of Engineering degree in Electrical/Electronic Engineering from University of Ado Ekiti, Ado, Ekiti State, in 2001, and a Master of Engineering degree from University of Benin, Benin, Edo State, Nigeria. His research areas include electric power system stability and protection, and renewable energy.