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Analytical Solution of the Schwartz - Moon Growth Option Model Revisited

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Abstract

In this work we revisited an earlier work, analytical solution of extended Schwartz and Moon growth option model, a model used for valuing a company, a particular case of a bank, the solution to the model proposed in the earlier work was represented and solved. The analytical problem presented in the earlier work was partitioned; an algorithm presented and solved using Monte Carlo simulation.

Keywords: Stochastic Differential Equation, Ito lemma, Simulation, bank value

1 Introduction

The framework of the present paper is based on the one presented in Owoloko [1], Schwartz and Moon [2] and a special case of Chang *et al* in [3]. The assumptions of the model proposed in [2] which also applies in [3] were enumerated in [1].
As stated in [1], the models in [2, 4], and those previously reported in literatures: [5], [6] and [7], where the model have been used, a discrete version of the continuous-time process is used to simulate the value of a company.
In [1], the mathematical formulation of the extended case was given and this led to the derivation of equation (25) of [1]. This equation is as a result of the application of Ito's lemma to the expression of bank value dynamics given as:

$$V \equiv V\left(L, \mu_L, D, \mu_D, \gamma, r, S, X, Y, p, t\right) \quad (1)$$

Where

$L =$ Bank loans

$\mu_L =$ Expected growth rate in loans

$D =$ Bank deposit

$\mu_D =$ Expected growth rate in deposit

$\gamma =$ Variable cost

$r =$ Interest rate

$S =$ Interest spread

$X =$ Loss-carry forward

$Y =$ Accumulated property, plant and equipment

$p =$ Cash balance

$t =$ Time

Other authors [9], [8] and [10] have also tried to model the value of banks. In particular, Owoloko *et al* gave the value of bank via the contingent claim approach [8].

2 Mathematical Formulation

In [1], the value of bank was given in equation (25), as

$$dV_t = \left\{ \begin{array}{l} \frac{\partial v}{\partial L} L\mu_t + \frac{\partial v}{\partial \mu_L} k\left(\overline{\mu_L} - \mu_L\right) + \frac{\partial v}{\partial D} D\mu_D + \frac{\partial v}{\partial \mu_D} k\left(\overline{\mu_D} - \mu_D\right) + \frac{\partial v}{\partial \gamma}\left(\bar{\gamma} - \gamma\right) \\ + \frac{\partial v}{\partial r} a^r \left(b^r - r_t\right) + \frac{\partial v}{\partial s} a^s \left(b^r - s_t\right) + \frac{\partial v}{\partial X}\left(R_t + Y_t + Dep_t - XCapX_t\right) + \frac{\partial v}{\partial \gamma} + \\ \frac{\partial v}{\partial P} + \frac{\partial v}{\partial t} + \frac{\partial^2 v}{2\partial L^2} L^2 \sigma_L^2 + \frac{\partial^2 v}{2\partial \mu_L^2} \gamma_L^2 + \frac{\partial^2 v}{2\partial D^2} D^2 \sigma_D^2 + \frac{\partial^2 v}{2\partial \mu_D^2} \gamma_D^2 + \frac{\partial^2 v}{\partial \gamma^2} \xi_t^2 + \\ \frac{\partial^2 v}{2\partial r^2} r\sigma_r^2 + \frac{\partial^2 v}{\partial s^2} s\sigma_s^2 + \frac{\partial v \partial v}{\partial L \partial \mu_L}\left(L\sigma_L \gamma_L\right)\varphi_{Lg} + \frac{\partial v \partial v}{\partial L \partial D}\left(L\sigma_L D\sigma_D\right)\varphi_{LD} + \\ \cdots + \frac{\partial v \partial v}{\partial \gamma \partial s}(\xi\sigma\sqrt{s})\varphi_{sr} + \frac{\partial v \partial v}{\partial r \partial s}\left(\sigma_r \sigma_s \sqrt{rs}\right)\varphi_{rs} \end{array} \right\} dt$$

$$+ \frac{\partial v}{\partial L} L\sigma_L dw_1 + \frac{\partial v}{\partial D} D\sigma_D dw_2 + \frac{\partial v}{\partial \mu_L} \gamma_L dw_3 + \frac{\partial w}{\partial \mu_D} \gamma_D dw_4 + \frac{\partial v}{\partial r} \sigma\sqrt{r} dw_5 \qquad (2)$$

$$+ \frac{\partial v}{\partial s} \sigma_s \sqrt{s} dw_6 + \frac{\partial v}{\partial \gamma} \xi dw_7$$

We concluded by saying that taking the integral of both sides of (2) with some necessary adjustments, the value of the bank can be found.

The new approach we adopted in finding solution to (2) is to partition the equation and then solve them separately. This approach was applied in [8]; that is, taking the integral of (2), we have:

$$V(t) = e^{-r(T-t)} \left\{ \frac{1}{N} \left[\left(p_1 - p_2\right) \sum_{i=0}^{N} L(t_i) + \left(p_4 - p_3\right) \sum_{i=0}^{N} D\left(t_i\right) - p_5 \sum \gamma(t_i) L(t_i) \right] + \Lambda \right\}$$

(3)

where

$N =$ Number of partitions

$$L(t_i) = L(t_{i-1}) \exp\left\{ \left[(\mu_L - \lambda\sigma_L) - \frac{\sigma_L^2}{2} \right] \right\} \Delta t_i + \sigma_L \in_i \sqrt{\Delta t_i}$$

$$D(t_i) = D(t_{i-1}) \exp\left\{ \left[(\mu_D - \lambda\sigma_D) - \frac{\sigma_D^2}{2} \right] \right\} \Delta t_i + \sigma_D \in_i \sqrt{\Delta t_i}$$

$$\gamma(t_i) = \bar{\mu} - \left(\bar{\mu} - \gamma\left(t_{i-1}\right)\right) e^{-k\Delta t_i} + \xi \in_i \sqrt{\frac{\left(1 - e^{-2k\Delta t_i}\right)}{2k}}$$

$$p_1 = (1 + r)(2 - r - \tau_c)$$

$p_2 = M(1+r)$

$p_3 = (s-1)(2+r-\tau_c)$

$p_4 = M(s-1)$

$p_5 = (1-\tau_c - M)$

$\Lambda = \left[\varpi(2+r-\tau_C) - F(1-\tau_C - M) - \tau_C Dep(T-t) - Capx(T-t) - M\varpi\right]$

3.0 The Simulation Algorithm

Equation (3) was implemented using the simulation algorithm below:
Set paths to value
Set period to a value
// $\in =$ random number

// $\lambda =$ market price

// $\Delta t =$ time interval

While Not EOF Do

For I = 1 to paths

For J = 1 to periods

Set time to J

Generate random number $\in$.

Multiply initial volatility loan growth rate by exponential (mean reversion-coefficient*time) and **store** in volatility rate for loan.

Call loan (J, I)// call function to compute loan // store the returned result of loan in **L.**

Multiply initial volatility deposit growth rate by exponential

(mean reversion coefficient*time) and store in volatility rate for deposit.

…

…

Call Deposit (J, I)// call function to compute deposit
Next J

Next I

Call Cash available (J, I), // call function to compute cash available, store the result in X.

Set M as multiplier for loan and deposit

Set C as addition of variable cost and fixed cost

$$V(t) = E_Q\left[\left(X(T) + M\Pi(T) - C(T)\right)\right]e^{-rT}$$

Print V as bank value.

END DO

End

FUNCTION loan (J, I)

//function to compute loan

// L = initial loan

// $\sigma_L =$ volatility of loan

// $\mu_L =$ growth rate in loan

Set $L(t_i) = L(t_{i-1})\exp\left\{\left[\left(\mu_L - \lambda\sigma\right) - \frac{\sigma_L^2}{2}\right]\right\}\Delta t_i \in_i \sqrt{\Delta t_i}$

RETURN L

FUNCTION Deposit

//Function to compute deposit

// D = Initial deposit

// $\sigma_D =$ volatility of deposit

// $\mu_D =$ growth rate of deposit

Set $D(t_i) = D(t_{i-1})\exp\left\{\left[\left(\mu_D - \lambda\sigma\right) - \frac{\sigma_D^2}{2}\right]\right\}\Delta t_i + \sigma_D \in_i \sqrt{\Delta t_i}$

RETURN D

FUNCTION Cash available (J, I)

// r = interest rate

// $\Pi(t) =$ bank revenue

// $Y(t) =$ after tax net income

// Compute depreciation

IF J = 1 **then** Dep = Dep **multiply** accumulated property

Else

Dep = Dep **multiply** (J-1, I)

End if

Set $X = (r+1)\Pi(t) + Y(t) + Dep - Capx(t)$

RETURN X (4)

4.0 Conclusion

In this paper, we modified the problem posed in [1], and a solution to the problem was solved by equation (3) using the simulation algorithm given in (4). With the formula given by (3) and the simulation algorithm given in (4), we can successfully estimate the value of a bank at an arbitrary time $t \in [0, T]$.

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APPENDIX

Symbol	**Definition**
$L(t)$	Bank loan at time t

$\mu(t)$	Growth rate of bank loan
M	Multiplier
$e^{-r\tau}$	Continuously compounded discount factor
$\delta_L(t)$	Volatility of bank loan at time t
$D(t)$	Bank deposit at time t
$\mu_D(t)$	Growth rate of bank deposit time t
$\delta_D(t)$	Volatility rate of bank deposit time t
W_1	Standard Brownian motion from the dynamics of loan
$\pi(t)$	Bank value time t
$\bar{\xi}$	Long-term average volatility of variable cost
W_2	Standard Brownian motion from the dynamics of deposit
W_3	Standard Brownian motion from the source of growth rate in loan
$\eta_L(t)$	Volatility of growth rate in loan at time t
$\eta_D(t)$	Volatility of growth rate in deposit at time t
k	Mean reversion coefficient
λ	Risk Premium
r_f	Risk free rate
β_k	Beta of the market
R_m	Market risk
$C(t)$	Total cost at time t
$\gamma(t)$	Variable cost at time t
F	Fixed cost
$\bar{\gamma}$	Long term average of variable cost
$\xi(t)$	Volatility of variable cost at time t
W_4	Standard Brownian motion associated with growth rate in deposit
W_5	Standard Brownian motion associated with variable cost
ϖ	Other sources of bank income
r	Interest on loan
s	Interest on deposit
$Y(t)$	After tax net income
$X(t)$	Cash balance at time
Dep	Depreciation
P	Accumulated property plant and equipment
$Capx$	Capital expenditure
DR	Percentage of depreciation
$V(t)$	Value of bank at an arbitrary time t

$V(0)$	Value of bank at present time t
E_Q	Equivalent martingale measure
τ_C	Corporate tax rate

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