TUTORIAL KIT
OMEGA SEMESTER

PROGRAMME: BANKING AND FINANCE

COURSE: BFN 425

QUANTITATIVE TECHNIQUE FOR FINANCIAL DECISIONS

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BFN 425: QUANTITATIVE TECHNIQUE FOR FINANCIAL DECISIONS

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TUTORIAL QUESTIONS

1. **Highlight five phases of quantitative analysis and explain briefly.**
2. **Model is a simplified representation of reality. Discuss?**
3. **Explain five types of quantitative models that can be employed in making financial decisions.**
4. **List and explain the two classifications of a model.**
5. **Quantitative models are of no relevance to financial decision making". Discuss and justifyyour position on this statement.**
6. **Explain the various reasons why quantitative approach might be useful in decision making process?**
7. **Express Commercial Bank Plc provides loans and advances to its customers within the available savings deposits (SD) and time deposits (TD) available to it. Each advance (A) absorbs 6 SD and 4TD and each loan (L) takes ISD and 2TD. The bank has only 8SD and 16TD at its disposal. Each overdraft contributes #8.00 profit and loan #10.00 profit.**
   Required:
   (i). Formulate a linear programming model for the bank to maximize the profit by allocating SD and TD between overdraft and loans.
   (ii). Using the simple algorithm, find an optimum solution to the problem in (a).
   (iii). If each of the constraints is increased by I unit, what will be the effect on profit?
8. **Differentiate between Normative and Descriptive Models.**
9. **Odua Electric Company produces two products P_1 and P_2. The products are produced and sold on a weekly basis. The weekly production cannot exceed 25 for product P_1 and 35 for product P_2 because of limited available facilities. The company employs total of 60 workers. Product P_1 requires two man-week of labour while P_2 requires one man-week of labour. Profit margin on P_1 is #60 and on P_2 is #40.**
   Required:
   Formulate this as a LP problem and solve for maximum profit graphically.
10. **Explain the various phases of quantitative analysis?**
11. **Discuss the nature and relevance of Decision Theory to financial decision making.**
12. **There are some MP models under conditions of certainty and some under condition of risk and uncertainty. Discuss and show their classification**

13. Assume Dr. Alex Ajayi has just bought a second hand deep-freezer from Dr. Roland Ikpefan to sell a new brand of iced tea called Icex in Canaanland. The selling price of icex is assumed to be N6000 and the cost price is N4800. Assume also that the daily demand for icex are 26,28,30 and 32 cartons with associated probabilities of 0.2, 0.4, 0.1, and 0.3 respectively. Before the purchase of the deep-freezer, Dr. Roland has been buying 28 cartons of icex for sale every morning, but Dr. Alex is not certain whether it is a good policy or that he should continue with the act.

Required:

As a quantitative financial expert, you are requested to advise Dr. Alex on the best policy of the appropriate number of cartons to be purchased each morning in order to make maximum returns, following these steps:

i. **Construct the payoff table for the transaction**
ii. **Determine the highest expected payoff**
iii. **Recommend the best act that will maximize his returns**

14. Explain any four features of LP

15. Explain Games Theory. Discuss five assumptions of games theory.

16. Explain any three advantages and disadvantages of LP

17. Consider the application of games theory to resolve the conflict situation between the NLC and the Federal Government over a new tax relief package (TRP), in which the motives of the two players are dichotomized. Each of the two participants has four major strategies to play the game, viz:

I – Threat and Strike

II – Lobbying and Pressurising Action

III – Arbitration and Court Action

IV – Tolerance and Reconciliation Action

The costs to the Federal Government are given for every pair of strategy choice as follows:

		Player B (Federal Government)				
	Strategies		**Strike and Threat**	**Lobbying and Pressuring**	**Arbitration and Court**	**Tolerance and Reconciliation**
			I	**II**	**III**	**IV**
Player A (NLC)	**Strike and Threat**	**I**	**20**	**15**	**12**	**35**
	Lobbying and Pressuring	**II**	**25**	**14**	**8**	**10**
	Arbitration and Court	**III**	**40**	**2**	**10**	**5**
	Tolerance and Reconciliation	**IV**	**-5**	**4**	**11**	**0**

Required:

i. **Find the value of the game.**

ii. **Determine the best strategy for NLC**

iii. **Determine the best strategy for the federal government**

iv. **State the implications of two-personszero-sum in the game**

v. **State two merits and two demerits of this game.**

18.Define games theory and explain any five of its assumptions

19. A company is considering investing in three kinds of projects. The company has an overriding objective to obtain a return of 15% under condition of minimum risk. The expected return for the first project is 8% while the second and third projects have expected returns of 12% and 20% respectively. Project one has standard deviation of its return as 0.02, project two as 0.0245 and project three as 0.0283. The company has a total of N1m to invest on the projects and it is assumed that the returns on the projects are statistically independent.

Required: Analyse and explain with the appropriate mathematical algorithm, how the company can minimize the risks in the face of the constraints.

20. Explain any four of the following concepts used in games analysis:
A) Strategy B) Value of the game C) Payoff Matrix D) Optimal strategy E) Two person zero-sum game.

ANSWERS

Question 1

i. *Formulation of the problem*: this is the first step in conducting quantitative analysis. it involves the identification and formulation of the actual problem to be investigated. The problem to be investigated must be clearly specified so that correct answer could be provided. The main objective of the study must be described to reflect an accurate description of the overall interest of the system. The system will be saved from producing inconsistent and unacceptable results, if the decision alternatives and constraints are taking into consideration.

ii. *Specification of a model*: the model should be a quantitative description of the objectives and constraints of the problem in terms of its decision variables. This type of model makes it possible for the operation research team to analyze the system and examine various alternatives without disrupting the behaviour of the entire system. As a framework of analysis, the model also helps to show what variables are relevant and what data are needed for the description of the system.

iii. *Estimation of the model*: this is the third phase of quantitative analysis. It involves model estimation for the purpose of providing accurate solutions to the problem. This involves fixing of data collected as input into the model. It is at this stage that the model is tested. Testing of a quantitative model may involve application of a well-defined optimization techniques through an optimum solution to the model may be determined. However since a mathematical model is a mere representation of the real problem, the resulting optimum solution, may not guarantee the best solution to the real system. But a good approximation to the optimum solution may suffice. In addition, the behaviour of the solution can further beanalysed by what is called sensitivity analysis. It explains what happens to the solution or how it will behave if deliberate changes are made in the system's parameter.

iv. *Validation of the model*: the next stage after securing the solution is to validate both the model and the solution obtained. A model may be adjudged valid if in the face of its limiting assumption and constraint, it gives a reliable or accurate prediction of the performance system. There are two common approaches which are relevant for assessing the validity of a model. One approach is by comparing the model's performance with the actual performance on the basis of past data. The second is to compare the model's performance with the behaviour of the system under the condition of no change. The predictive ability of the model can therefore be ascertained on the basis of these two approaches.

v. *Implementation of output*: this is the last stage of quantitative analysis and it involves the implementation of the results obtained from the model tested. This requires that the analyst should translate the model's output in unambiguous manner to operating procedure for the understanding of the party responsible for the operation of the system. After the implementation of the output, the response of the system to the changes should be objective and necessary adjustments made if need be. It suggests that there should be cooperation between operation research analyst and the party involved in operating and managing the system.

Question 3

i. ***Iconic model***: They are visual models of real life situations or problems in form of symbols, pictures objects, diagrams etc. Iconic model is a look alike representation of some specific entity.

ii. ***Analogue model***: This model is used to simplify reality through the use of analogy. Analogue model uses one type of behaviour of variable or properties to represent another type. It represents the relationship between two variables by a line on a chart and as a result conveys the relationship between the variables in an efficient and convenient way. Graphs are the most frequently used analogue models.

iii. ***Simulation model***: Simulation model consist of a series of equations and formula arranged systematically so that they "behave" in a similar manner to the real system being investigated. This type of model is relevant where there are no appropriate mathematical models to capture a problem.

iv. ***Mathematical or symbolic models***: they explain the behaviour of the real world by means of mathematical and statistical notation, symbols, equations and formulae. Symbolic models are often used whenever the reality is too complex or too abstract to be portrayed through an iconic model or analogue model

v. ***Heuristic Model*****:** refers to techniques based on experience for various tasks such as research, problem solving, discovery and learning. These models attempt to show how to arrive at a workable solution to a problem or a better option to a current practice by following a set of intuitive rules or direction.

Question 5

Quantitative models are relevant to financial decision making because it can be applied in the following areas of financial operations:

i. ***Cash management***: Quantitative technique like linear programming, can be used to achieve the objective by determining the proportion of total funds that should be allocated to the various units, given the available constraints.

ii**.** ***Capital budgeting*****:** Quantitative techniques can be used in the process of investing an organization's current fund most efficiently in long term activities in anticipation of an expected flow of future benefits over a series of years.

iii**.** ***Inventory control*****:** The Economic Order Quantity (EOQ) technique used in quantitative analysis for inventory management can help the financial managers to develop optimal inventory policies and reduce the investment in inventories or their waste.

iv. ***Determination of an organization's optimal capital structure***: It is also useful in determining an organization's optimal capital structure. Capital structure involves the analysis of the combination of a company's selected bonds, debentures, preferred stocks and common stock.

v. ***Investment Analysis and Portfolio Management***: Quantitative models can also be applied in portfolio management in order to get an optimum or efficient portfolio.

Question 7

(i) State the constraints and objective functions. The portfolio must satisfy the inequalities.

Let: X_1 = Advance
X_2 = Loan

Constraint Function: $6X_1 + 1X_2 \leq 8$ (SD Constraint)
$4X_1 + 2X_2 \leq 16$ (TD Constraint)

Objective Function: $\not{Z} = 8X_1 + 10X_2$ (Profit Objective Function)

Hence: Maximize the objective: $\not{Z} = 8X_1 + 10X_2$
Subject to the constraints: $6X_1 + 1X_2 \leq 8$
$4X_1 + 2X_2 \leq 16$

State the constraints in standard form by turning inequalities into equations.

$6X_1 + 1X_2 + 1S_1 + 0S_2 + 0\not{z} = 8$
$4X_1 + 2X_2 + 0S_1 + 1S_2 + 0\not{z} = 16$
$8X_1 + -10X_2 + 0S_1 + 0S_2 + 1\not{z} = 0$

State the coefficient of variables (values):

8 = 6 1 1 0 0
16 = 4 2 0 1 0
0 = -8 -10 0 0 1

This is presented in the first simplex tableau, showing coefficient and variables.

Tableau 1

leave

					enter	●	●		
	Row no	Variables	Values	X_1	X_2	S_1	S_2	$\not{Z}$	How much
SD	1	S_1	8	6	1	1	0	0	8/1 = 8
TD	2	S_2	16	4	2	0	1	0	16/2 =8
Index	3	$\not{Z}$	0	-8	-10	0	0	1	

The Pivot Column = X_2
The Pivot Row = TD
The Pivot Coefficient = 2

Next: Fill the rows in a second table by using the following formulas:

New 2nd Row = Old Pivot Row / Pivot Coefficient = 16 4 2 0 1 0 / 2

(8 2 1 0 0.5 0)

New 1st Row= Old first row – 1 (new second row)

= 8 6 1 1 0 0 - 1 (8 2 1 0 0.5 0)

= 8 – 8, 6 – 2, 1 – 1, 1 – 0, 0 - 0.5, 0 – 0

= 0 4 0 1 -0.5 0

New 3rd Row = Old index row – (-10) (new second row)

= 0 -8 -10 0 0 1 – (-10) (8 2 1 0 0.5 0)

= 0 -8 -10 0 0 1 + (80 20 10 0 5 0)

= 0 + 80, -8 + 20, -10 + 10, 0 + 0, 0 + 5, 1+ 0

= 80 12 0 0 5 1

Table 2

				enter	●	●			
	Row no	Variables	Values	X_1	X_2	S_1	S_2	Z	How much
Advance	1	S_1	0	4	0	1	-0.5	0	
Loans	2	X_2	8	2	1	0	0.5	0	
Index	3	Z	80	12	0	0	5	1	

(ii). The optimum solution can be obtained if the bank:

- N0m on advance
- N8m on Loans and
- The expected profit is N80m

(iii). If loans constraint is increased by 1 unit, the profit will also increase by its shadow price of N5m. If advance constraint is increased by 1 unit, there will be no increase in profit i.e N0m shadow price as shown in the index row of table 2.

Question 9

Step 1: Define the decision variables

Let X_1 = No. of units of P_1 to be produced

X_2 = No. of units of P_2 to be produced

Maximize the total Naira contribution.

$Z = ₦60X_1 + ₦40X_2$

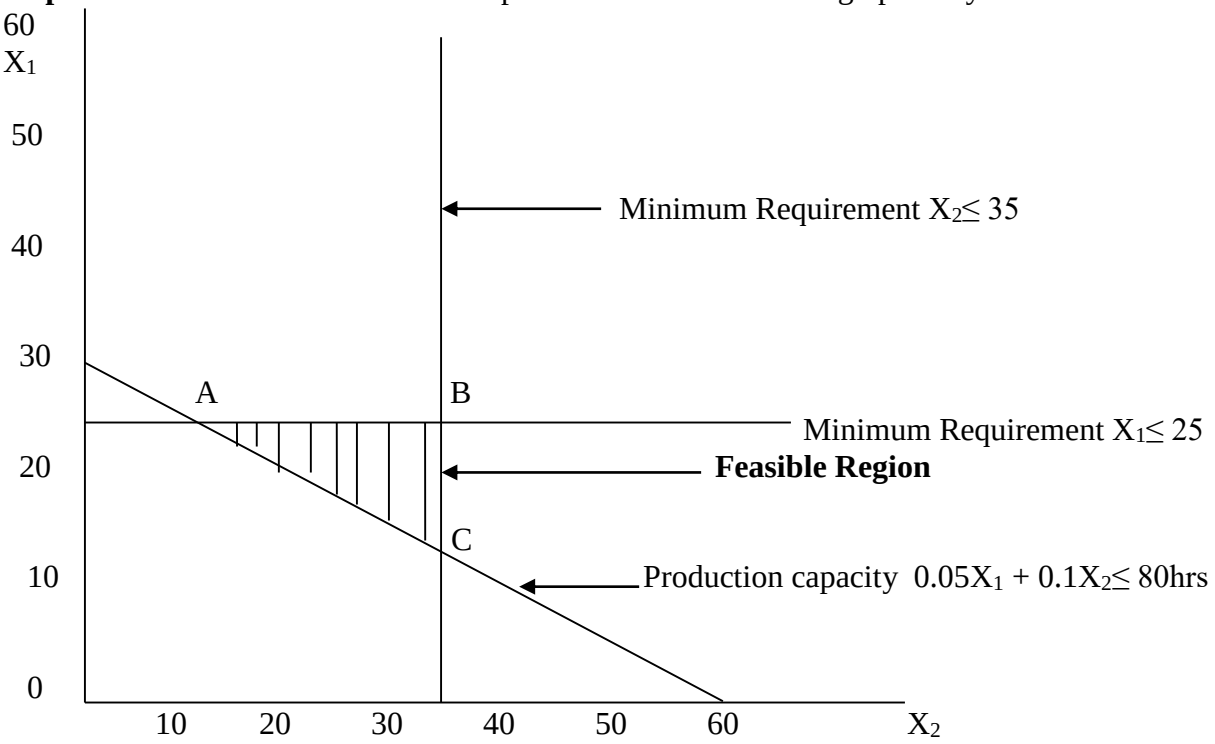
Subject to the constraint equations:

Production capacity: $2X_1 + 1X_2 \leq 60$ hours

Minimum Requirement: $X_1 \leq 25$ units

$X2 \leq 35$ units

Step 2: The solution to the formulated problems can be obtained graphically as follows:



Production Capacity: $X_1 = 60/2 = 30$

$X_2 = 60/1 = 60$

Step 3: The result of the solution can be shown in a table as follows:

Production points	X_1 (Units)	X_2 (Units)	Value of the obj function (Naira)
A	25	10	1900
B	25	35	2900
C	15	35	2300

Solving for the value of the objective function we will use $Z = ₦60X_1 + ₦40X_2$

Point		X_1		X_2	
A	=	₦ 60 x 25	+	₦ 40 x 10	
		₦ 1200	+	₦1200	= ₦ 190
B	=	₦ 60 x 25	+	₦ 40 x 35	
		₦ 800	+	₦ 1800	= ₦ 2900

C = ₦ 60 x 15 + ₦ 40 x 35
₦ 1400 + ₦ 1200 = ₦ 2300

Decision: The objective function is maximized at point B (25 units of P_1 and 35units of P_2) where it equals ₦ 2900. Hence, the firm should produce 25 units of product P_1 and 35units of P_2.

Question 11

Nature and Relevance of Decision Theory to Financial Decision Making:

i. Decision analysis refers to application of quantitative data and rational approach to problem of decision making under uncertainty.
ii. It provides the financial manager with the appropriate tools for determination of optimum decisions in the face of uncertainty.
iii. It has wide applicability and relevance to the field of finance and other studies

Question 13

i. The pay-off table is constructed as follows:

			13 packs			14 packs			15 packs			16 packs		
Outcome	Demand	Prob	Sales	Pay-off	Prob x pay-off	Sales	Pay-off	Prob x pay-off	Sales	Pay-off	Prob x pay-off	Sales	Pay-off	Prob x pay-off
X_1	26	0.2	26	31200	6240	26	21600	4320	26	12000	2400	26	3000	600
X_2	28	0.4	26	31200	12480	28	33600	13440	28	24000	9600	28	15000	6000
X_3	30	0.1	26	31200	3120	28	33600	3360	30	36000	3600	30	27000	2700
X_4	32	0.3	26	31200	9360	28	33600	10080	30	36000	10800	32	39000	11700
EXPECTED PAY-OFF					**30840**			**31200**			**26400**			**21000**

ii. The highest expected pay-off is N31200 which corresponds to the act of purchasing 28 cartons every morning**.**

iii. It is recommended that Dr. Alex Ajayi should purchase 28 packs of 'icex' for sale every morning.

Question 15

Games theory deals with analysis of decision making when two or more intelligent and rational opponents are involved under conditions of conflict and competition.

Assumptions of Game Theory:

- Each player aims at maximising gains and minimizing losses.
- Each player has a definite set of probable courses of action or strategies available to him.
- Each player has the knowledge of both strategies
- Each player behaves naturally
- The first player's gains are the second player's losses and vice versa.

Question 17

		Player B (Federal Government)						
	Strategies		Strike and Threat	Lobbying and Pressuring	Arbitration and Court	Tolerance and Reconciliation	**Row minimum**	**Maximin**
Player A (NLC)			**I**	**II**	**III**	**IV**		**12**
	Strike and Threat	**I**	20	15	12	35	12	
	Lobbying and Pressuring	**II**	25	14	8	10	8	
	Arbitration and Court	**III**	40	2	10	5	2	
	Tolerance and Reconciliation	**IV**	-5	4	11	0	-5	
	Col. maximum		40	15	12	35		
	Minimax	**12**						

Saddle Point exists: maximin = minimax = 12

i. The value of the game = 12
ii. The best strategy for NLC is strategy I, Strike and Threat.
iii. The best strategy for Federal Government is strategy III which is Arbitration and Court.
iv. The implication of two-persons-zero sum game is that the gains of NLC in the game are the losses of the Federal Government.

Question 19

Non-Linear Mathematical Programming Model

Let X_1, X_2, X_3 represent projects 1, 2, 3 respectively.

Total Fund Constraint

$X_1+X_2+X_3 = \#\ 1000{,}000$

$X_1+X_2+X_3 = 1000$

The Expected returns Constraint

$0.08X_1 + 0.12X_2 + 0.2X_3 = 0.15\ (1000)$

$0.08X_1 + 0.12X_2 + 0.2X_3 = 150$

The risks of the Portfolio (Objective Function)

The risks is measured by the variance i.e. $(S.D)^2$

Note: The objective is to minimize the risk.

$Z = (0.02X_1)^2 + (0.0245X_2)^2 + (0.0283X_3)^2$

$= 0.0004X_1^2 + 0.0006X_2^2 + 0.0008X_3^2$

Step 1: Modify the Constraints:

From the first constraint we obtain

$X_1 = 1000 - X_2 - X_3$..... (1)

Step 2: Substitute equation (1)for (X_1) in the Second Constraint

$0.08(1000 - X_2 - X_3) + 0.12(X_2) + 0.2(X_3) = 150$

$80 - 0.08X_2 - 0.08X_3 + 0.12X_2 + 0.2X_3 = 150$

$0.12X_2 - 0.08X_2 + 0.2X_3 - 0.08X_3 = 150 - 80$

$0.04X_2 + 0.12X_3 = 70$

$0.04X_2 = 70 - 0.12X_3$

$$X_2 = \frac{70 - 0.12X_3}{0.04}$$

$X_2 = 1750 - 3X_3$.....(2)

Step 3: Substitute equation (2) for X2in equation (1)

$X_1 = 1000 - X_2 - X_3$

$X_1 = 1000 - (1750 - 3X_3) - X_3$

$X_1 = -750 + 2X_3$

$X_1 = 2X_3 - 750$

Step 4: Substitute the values for X_1 and X_2 into the objective function

Recall that $Z = 0.0004X_1^2 + 0.0006X_2^2 + 0.0008X_3^2$

$= 0.0004\ (2X_3 - 750)^2 + 0.0006\ (1750 - 3X_3)^2 + 0.0008X_3^2$

$=0.0004\ (4X_3^2 - 3000X_3 + 562{,}500) + 0.0006\ (9X_3^2 - 10500X_3 + 3{,}062{,}500) + 0.0008X_3^2$

$= 0.0016X_3^2 - 1.2X_3 + 225 + 0.0054X_3^2 - 6.3X_3 + 1838 + 0.0008X_3^2$

Collect like terms;

$\not{Z} = 0.0016X_3^2 + 0.0054X_3^2 + 0.0008X_3^2 - 1.2X_3 - 6.3X_3 + 225 + 1838$

$= 0.0078X_3^2 - 7.5X_3 + 2063$ (Quadratic problem)

Step 5: Solvefor X_3 using the Quadratic Formula

$\not{Z} = 0.0078X_3^2 - 7.5X_3 + 2063$

$a = 0.0078$, $b = -7.5$ and $c = 2063$

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{7.5 + \sqrt{(-7.5)^2 - 4\,(0.0078) \times 2063}}{2 \times 0.0078}$$

$$\frac{7.5 + \sqrt{56.25 - 64.36}}{0.0156}$$

$$\frac{7.5 + \sqrt{-8.12}}{0.0156}$$

$$\frac{7.5 + 0}{0.0156}$$

$X_3 = 480.77$

$X_2 = 1750 - 3X_3$

$= 1750 - 3(480.77)$

$= 1750 - 1442.31$

$= 307.69$

$X_1 = 2X_3 - 750$

$= 2\,(480.77) - 750$

$= 211.54$

$X_1 = \#211{,}540$, $X_2 = \#307{,}690$ and $X_3 = \#480{,}770$

Expected Return

Stock 1 $= 0.08X_1 = 0.08\,(211{,}540) = 16923$

Stock 2 $= 0.12X_2 = 0.12\,(307{,}690) = 39923$

Stock 3 = $0.12X_3$ = 0.12 (480,770) = 96154
#150,000

DECISION:
The investor should invest in #211,540 project 1, #307,690 in project 2 and #480,770 in project3 to be able to achieve the expected return of 15% on the #1,000,000 = #150,000