Solutions of the Schrödinger equation with inversely quadratic effective plus Mie-type potential using Nikiforov-Uvarov method

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Solutions of the Schrödinger Equation with Inversely Quadratic Effective Plus Mie-Type Potential using Nikiforov-Uvarov Method

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Abstract. The Schrödinger equation with the interaction of inversely quadratic effective and Mie-type potential has been solved for any angular momentum quantum number \( l \) using the Nikiforov-Uvarov method. The bound state energy eigenvalues and the corresponding un-normalized eigenfunctions are obtained in terms of the Laguerre polynomials. Several cases of the potential are also considered and their eigen values obtained.

Keywords: Schrödinger equation; inversely quadratic effective potential; Mie-type potential; Nikiforov-Uvarov method
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INTRODUCTION

The Schrödinger equation (SE) is a very important equation for non-relativistic quantum mechanical systems containing atomistic particles like the electrons. These electrons revolve around their orbits forming a bound state system. The solutions of the SE will therefore add impetus to our understanding of these systems. It is observed that bound state solutions of the SE equation are only possible for some potentials of physical interest [1-5]. The problem of obtaining exact or approximate solutions of the Schrödinger equation for a number of special potentials have also been speculated [6-11]. Some of these potentials are known to play very important roles in many fields of Physics such as Molecular Physics, Solid State and Chemical Physics [8].

The purpose of the present work is to extend our earlier paper published in [12] for a special case of the potential with \( a = 0 \) to obtain a mix potential involving inversely quadratic effective and Mie-type potential (IQEMP) of the form

\[ V(r) = \frac{D}{r^2} e^{br} + \left( -\frac{A}{r} + \frac{B}{r^2} + C \right) \]

in the Schrodinger formalism. We have also plotted the potential as a function of the inter-nuclear distance in Figure 1 and also done some numerical computation of the energy eigenvalues which were not done in reference [12]. In the potential above, \( A, B, C, D \) are constants that relate to the coulombic interactions between two electrons. The results of the work will therefore be applied to electrons moving in such potential.

This potential can be written as

\[ V(r) = \frac{B+D}{r^2} - \frac{A+Db}{r} + (C+Db^2), \]  \hspace{1cm} (1)

where \( r \) represents the internuclear distance, \( b \) is a constant. Equation (1) is then amenable to Nikiforov-Uvarov method. The effective potential between atomic electrons was suggested by Bransden and Joachain [13] in their book on atomic electrons. However, not much has been achieved in the area of solving the radial Schrödinger equation for any angular momentum quantum number, \( l \) with IQEMP using Nikiforov-Uvarov method in the literature.

Overview of the Nikiforov-Uvarov method

The overview of the Nikiforov-Uvarov (NU) method has already been given in detail by us [12].
The Schrödinger Equation

In spherical coordinate, Schrödinger equation with the potential \( V(r) \) is given as [14]

\[
\frac{-\hbar^2}{2\mu} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right] \psi(r, \theta, \phi) + V(r) \psi(r, \theta, \phi) = E \psi(r, \theta, \phi).
\]  

(2)

Using the common ansatz for the wave function:

\[
\psi(r, \theta, \phi) = \frac{R(r)}{r} Y_{l m}(\theta, \phi)
\]

(3)

in equation (8) we get the following set of equations:

\[
\frac{d^2 R_n(r)}{d r^2} + \frac{2 \mu}{\hbar^2} \left[ E - V(r) - \frac{\lambda^2}{2 \mu r^2} \right] R_{nl}(r) = 0
\]

(4)

\[
\frac{d^2 \theta_{ml}(\theta)}{d \theta^2} + \cot \theta \frac{d \theta_{ml}(\theta)}{d \theta} \left( \lambda - \frac{m^2}{\sin^2 \theta} \right) \theta_{ml}(\theta) = 0
\]

(5)

\[
\frac{d^2 \phi_{m}(\phi)}{d \phi^2} + m^2 \phi_{m}(\phi) = 0
\]

(6)

where \( \lambda = l(l + 1) \) and \( m^2 \) are the separation constants. \( Y_{l m}(\theta, \phi) = \theta_{ml}(\theta) \phi_{m}(\phi) \) is the solution of equations (5) and (6) and their solutions are well known as spherical harmonic functions [15].

\[FIGURE 1.\] The plot of IQE + Mie type potential for \( A=1, B=-2, C=2, D=-5 \) and \( b=0.002 \)

Solutions to the radial equation

Equation (4) is the radial part of the Schrödinger equation which we are interested in solving. Equation (4) together with the potential in equation (1) and with the transformation \( z = r^2 \) yields the following equation:
where the radial wave function is $R(z)$ and

$$\alpha = \frac{2\mu(E-C-Bb^2)}{\hbar^2}, \quad \beta = \frac{2\mu(A+Db)}{\hbar^2}, \quad \gamma = \frac{2\mu(D)}{\hbar^2} + l(l + 1).$$

Following the method in our paper [12] we obtain the solutions of the SE equation with the mixed potential as

$$E = C + Db^2 - \frac{\mu(A+Db)^2/2\hbar^2}{{n+\frac{1}{2}+\frac{2\mu B}{\hbar^2}+(\frac{1}{2})^2}},$$

The radial wave function is obtained as

$$R(z) = N_n z^{-(1+\sqrt{1+4\gamma})/2} e^{-\sqrt{\alpha z}} L_n^{1+4\gamma}(2\sqrt{\alpha z}),$$

where $N_n$ is the normalization constant.

**DISCUSSION**

Having obtained the energy eigenvalues and corresponding eigenfunctions of the IQEMP we now consider the following cases of the potential.

**Case 1:** If we set the parameters, $B = C = D = 0, A = z\beta^2$, it is easy to show that equation (9) reduces to the bound state energy spectrum of a particle in the Coulomb potential, i.e., $E_C = -2^2\mu e^4/2\hbar^2 n_p^2$ where $n_p = n + l + 1$, is the principal quantum number.

**Case 2:** Similarly, if we set $D = 0, A \neq 0, B \neq 0, C \neq 0$ equation (9) results in the bound state energy spectrum of a vibrating-rotating diatomic molecule subject to the Mie-type potential as follows:

$$E_M = C - \frac{\mu(A)^2/2\hbar^2}{{n+\frac{1}{2}+\frac{2\mu B}{\hbar^2}+(\frac{1}{2})^2}},$$

**Case 3:** If we set $C = D = 0, A \neq 0, B \neq 0$ we obtain the energy spectrum of the Kratzer-Feus potential as

$$E_{KF} = -\frac{\mu(A)^2/2\hbar^2}{{n+\frac{1}{2}+\frac{2\mu B}{\hbar^2}+(\frac{1}{2})^2}},$$

**Case 4:** If we set $B = C = 0, b \neq 0, A \neq 0, D \neq 0$, we have the energy spectrum of the coulomb plus inversely quadratic effective potential as

$$E_{IQEC} = Db^2 - \frac{\mu(A+Db)^2/2\hbar^2}{{n+\frac{1}{2}+\frac{2\mu B}{\hbar^2}+(\frac{1}{2})^2}},$$

Equations (11) and (12) are similar to the ones obtained in reference [16].
Table 1. Energy spectrum for IQE + Mie-Type potential for \( A=1, B=-2, C=2, D=-5 \) with different values of \( b = 0.01, 0.02, 0.005 \) and 0.1

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<th>(E_n^e) for (b=0.002)</th>
<th>(E_n^e) for (b=0.005)</th>
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**CONCLUSION**

The bound state solutions of the Schrodinger equation have been obtained for the interaction of inversely quadratic effective plus Mie-type potential using Nikiforov-Uvarov method. With appropriate choice of parameters, the potential reduces to Coulomb, Mie-type, Kratzer-Feus and Coulomb plus inversely quadratic effective potentials and their energy eigenspectra obtained.

**REFERENCES**