COVENANT UNIVERSITY
NIGERIA

TUTORIAL KIT
OMEGA SEMESTER

PROGRAMME: ECONOMICS

COURSE: ECN 225
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The contents of this document are intended for practice and leaning purposes at the undergraduate level. The materials are from different sources including the internet and the contributors do not in any way claim authorship or ownership of them. The materials are also not to be used for any commercial purpose.
1. A firm faces the production function \( Q = 120L + 200K - \frac{1}{2}L^2 - 2K^2 \) for positive values of \( Q \). It can buy \( L \) at \( \text{₦5} \) a unit and \( K \) at \( \text{₦8} \) a unit and has a budget of \( \text{₦70} \). What is the maximum output it can produce?

2. Write out the properties of Definite Integrals with at least two examples each.

3. Evaluate \( \int_0^4 \left( \frac{1}{1+x} + 2x \right) \, dx \)

4. Find \( \int \left( 2e^{2x} + \frac{14x}{7x^2+5} \right) \, dx \)

5. Give examples of Improper Integrals with infinite limits of integration and Improper Integrals with infinite integrands

6. If the marginal cost of a firm is given by the following function of output, \( C'(Q) = 2e^{0.2q} \) and the fixed cost is \( C_F = 90 \). Find the total cost \( C(Q) \).

7. Given the MC function of a firm to be \( MC = \frac{dc}{dq} = 1.2q^2 - 18q + 100 \). Derive the total cost function of the firm.

8. A firm faces the production function \( Q = 12K^{0.4}L^{0.4} \) and can buy the inputs \( K \) and \( L \) at prices per unit of \( \text{₦40} \) and \( \text{₦5} \) respectively. If it has a budget of \( \text{₦800} \) what combination of \( K \) and \( L \) should it use in order to produce the maximum possible output?

9. Differentiate the following functions.
   
   \( y = 4x^2 + 2x^3 - x^4 + 0.1x^5 \)

   \( y = 20 + 4x - 0.5x^2 + 0.01x^3 \)

   \( y = 25 - 0.1x^2 + 2x^{0.3} \)

10. Explain any \textbf{FIVE} rules of differentiation.

11. Find \( \int e^{4x+3} \, dx \)

12. Evaluate \( \int e^{x^3+x^2+1} \, dx \)

13. Find \( \int (3x^5 + 2x^3 + 1) \, dx \)

14. Find the point of inflection if any, of the function \( y = x^3 + 3x^2 + 3x \)

15. Ascertain whether the function \( y = f(x) = x^3 - 6x^2 + 12 \) has a point of inflection and determine it.

16. With the method of Lagrange multiplier,

   \[
   \text{Minimize } z = x^2 - xy + \frac{3}{2}y^2 \\
   \text{Subject to: } x + 2y = 3
   \]

17. Optimize the function \( z = 4x + 6y - x^2 - y^2 \)
A firm faces the production function $Q = 20K^{0.4}L^{0.6}$. It can buy inputs $K$ and $L$ for ₦400 a unit and ₦200 a unit respectively. What combination of $L$ and $K$ should be used to maximize output if its input budget is constrained to ₦6,000?
Solution
1. \( MP_L = \frac{dQ}{dL} = 120 - 2L \quad MP_L = \frac{dQ}{dL} = 200 - 4K \)

For optimal input combination,
\[
\frac{MP_K}{P_K} = \frac{MP_L}{P_L}
\]
Therefore, substituting \( MP_K \) and \( MP_L \) and the given input prices,
\[
\frac{120 - 2L}{8} = \frac{200 - 4K}{8}
\]
\[
8(120 - 2L) = 5(200 - 4K)
\]
\[
960 - 16L = 1000 - 20K
\]
\[
20K = 40 + 16L
\]
\[
K = 2 + 0.8L
\]
Substituting (1) into the budget constraint
\[
5L + 8K = 70
\]
gives
\[
5L + 8(2 + 0.8L) = 70
\]
\[
5L + 16 + 6.4L = 70
\]
\[
11.4L = 54
\]
\[
L = 4.74 \text{ (to 2 dp)}
\]
Substituting this result into (1)
\[
K = 2 + 0.8(4.74) = 5.79
\]
Therefore maximum output is
\[
Q = 120L + 200K - L2 - 2K2
\]
\[
= 120(4.74) + 200(5.79) - (4.74)^2 - 2(5.79)^2
\]
\[
= 1,637.28
\]
This technique can also be applied to consumer theory, where utility is maximized subject to a budget constraint.

2. Find \( \int (2e^{2x} + \frac{14x}{7x^2 + 5}) \, dx \)

Integrating separating, \( 2e^{2x} \) is in the form \( f'(x) \cdot e^{f(x)} \)
Thus, \( \int 2e^{2x} \, dx = e^{2x} + C_1 \)
\[
\int \frac{14x}{7x^2 + 5} \, dx \text{ is in the form } \frac{f'(x)}{f(x)} = \ln(7x^2 + 5) + C_2
\]
Thus, \( \int (2e^{2x} + \frac{14x}{7x^2 + 5}) \, dx = e^{2x} + C_1 + \ln(7x^2 + 5) + C_2 \)
\[
= e^{2x} + \ln(7x^2 + 5) + C
\]

3. Recall TC = VC + FC
Total cost function is the integral of the marginal cost function.
\[
\int 2e^{0.2Q} \, dq = 2e^{0.2Q}_{0.2} + C = 2 - \frac{1}{0.2} (e^{0.2Q}) + C
\]
\[
= 10e^{0.2Q} + C
\]
Given \( C_F \) as 90 implies total cost when \( Q = 0 \).
Setting \( Q = 0 \) yields \( 10e^{0.2(0)} + C = 90 \)
Thus, \( C = 90 - 10 = 80 \)

Hence, the total cost function \( C(Q) = 10e^{0.2Q} + 80 \)

4. \( MP_L = \frac{dQ}{dL} = 12K^{0.4}L^{-0.4} \quad MP_K = \frac{dQ}{dK} = 8K^{-0.6}L^{0.6} \)

Optimal input mix requires;

\[
\frac{MP_K}{P_K} = \frac{MP_L}{P_L}
\]

Therefore,

\[
\frac{12K^{0.4}L^{-0.4}}{200} = \frac{8K^{-0.6}L^{0.6}}{400}
\]

Cross multiplying yields;

\[
4,800K = 1,600L
\]

\( 3K = L \)

Substituting this result into the budget constraint

\( 200L + 400K = 6,000 \)

gives

\( 200(3K) + 400K = 6,000 \)

\( 600K + 400K = 6,000 \)

\( 1,000K = 6,000 \)

\( K = 6 \)

Therefore,

\( L = 3K = 18 \)

5. Total cost function of the firm is the integral of the marginal cost. Thus,

\[
TC = \int MC \, dq = \int 1.2q^2 - 18q + 100 \, dq
\]

\[
= (1.2) \frac{q^3}{3} - \frac{18q^2}{2} + 100q + C
\]

\[
= 0.4q^3 - 9q^2 + 100q + C
\]

The value of \( C \) is dependent on the fixed cost and it is necessarily positive. i.e. \( C > 0 \)

6. The problem is to maximize the function \( Q = 12K^{0.4}L^{0.4} \) subject to the budget constraint

\( 40K + 5L = 800 \) (1)

The theory of the firm tells us that a firm is optimally allocating a fixed budget if the last ₦1 spent on each input adds the same amount to output, i.e. marginal product over price should be equal for all inputs. This optimization condition can be written as;

\[
\frac{MP_K}{P_K} = \frac{MP_L}{P_L}
\]

(2)

The marginal products can be determined by partial differentiation:

\[
MP_K = \frac{dQ}{dK} = 4.8K^{-0.6}L^{0.4}
\]

(3)

\[
MP_L = \frac{dQ}{dL} = 4.8K^{0.4}L^{-0.6}
\]

(4)

Substituting (3) and (4) and the given prices for \( P_K \) and \( P_L \) into (2)

\[
\frac{4.8K^{-0.6}L^{0.4}}{40} = \frac{4.8K^{0.4}L^{-0.6}}{5}
\]

Dividing both sides by 4.8 and multiplying by 40 gives;

\[
K^{-0.6}L^{0.4} = 8K^{0.4}L^{-0.6}
\]
Multiplying both sides by $K^{0.6}L^{0.6}$ gives;
$L = 8K$  \hspace{1cm} (5)
Substituting (5) for $L$ into the budget constraint (1) gives;
$40K + 5(8K) = 800$
$40K + 40K = 800$
$80K = 800$
Thus the optimal value of $K$ is
$K = 10$
and, from (5), the optimal value of $L$ is
$L = 80$