COVENANT UNIVERSITY
NIGERIA

TUTORIAL KIT
OMEGA SEMESTER

PROGRAMME: PHYSICS

COURSE: PHY 221

DISCLAIMER

The contents of this document are intended for practice and leaning purposes at the undergraduate level. The materials are from different sources including the internet and the contributors do not in any way claim authorship or ownership of them. The materials are also not to be used for any commercial purpose.

PHY 221: Fluid Dynamics

CONTRIBUTOR: Dr. Akinyemi M.L.

1. State the aspects of fluid mechanics which differentiate it from solid mechanics
2. Describe with the aid of diagrams uniform and non-uniform velocity profiles in fluid flow .
3. Categorize fluids based on their coefficient of dynamic viscosity μ and illustrate this with a diagram
4. Differentiate between Newtonian and Non-Newtonian fluids.
5. Given the equation $\tau = A + B\left(\dfrac{\delta u}{\delta y}\right)^{n}$ to represent both types of fluids, what are the values of A, B and n for Newtonian fluids
6. The following is a table of measurement for a fluid at constant temperature. Determine the coefficient of dynamic viscosity μ of the fluid

du/dy (s^{-1})	1. 0.0	2. 0.3	3. 0.5	4. 0.7	5. 0.9
τ (Nm^{-2})	6. 0.0	7. 1.5	8. 2.4	9. 3.6	10. 4.4

7. Using dimension analysis, derive the equation for μ

8. What are the distinct characteristics of liquids and gases?

9. Explain why the viscosity of a liquid decreases with temperature rise while that of a gas increases with temperature rise

10. Define and deduce the dimensions for the following parameters: mass density, specific weight and relative density

11. The density of oil is 850 kg/m³. Find its relative density and Kinematic viscosity if the dynamic viscosity is 5 x 10⁻³ kg/ms.

12. The velocity distribution of a viscous liquid flowing over a fixed plate is given by $u = 0.68y - y^2$ (u is velocity in m/s and y is the distance from the plate in m). What are the shear stresses at the plate surface   and at y=0.34m? (Dynamic viscosity = 0.9 Ns/m²).

3

13. What are the properties of a static fluid?

14. Prove that for a certain element of fluid, pressure at a point P is the same in all directions.

15. Use the continuity principle to determine the fluid flow velocities in pipes coming from a Y-junction. If pipe 1 diameter = 50mm, mean velocity 2m/s, pipe 2 diameter 40mm takes 30% of total discharge and pipe 3 diameter 60mm. What are the values of discharge and mean velocity in each pipe?

16. What are two dimensional fluid flow? With diagram describe stream lines and enumerate their properties

17. In a fluid flow, show that the Bernoulli's equation relating the states at two points along a streamline, is given by $\dfrac{p_1}{\rho g} + \dfrac{u_1^2}{2g} + z_1 = \dfrac{p_2}{\rho g} + \dfrac{u_2^2}{2g} + z_2$. Where p, u and z represent the pressure, velocity and height of the fluid at the two points, g is gravitational acceleration and ρ is density of the fluid.

18. Illustrate the function of a pitot tube with a diagram, showing the stagnation point. Derive the equation for pressure at the stagnation point.

19. The entry pressure of fluid flowing through a pitot tube is 1250 Pas, the pressure is 1710 Pas. Given the density of the fluid is 850kg/m³, Determine the entry velocity of the fluid into the pitot tube and the stagnation head.

20. A Venturimeter with an entrance diameter of 0.3m and a throat diameter of 0.2m is used to measure the volume of gas flowing through a pipe. The discharge coefficient of the meter is 0.96. Assuming the specific weight of the gas to be constant at 19.62 N/m³, calculate the volume flowing when the pressure difference between the entrance and the throat is measured as 0.06m on a water U-tube manometer.

SOLUTION

Q1.

There are few aspects of fluid mechanics which differentiate it from solid mechanics:

(1) In fluids we usually deal with *continuous* streams of flow without a beginning or an end. In solids we only consider individual elements.

(2) A fluid cannot resist the deformation force, it moves and *flows* under the action of the force. A solid can resist a deformation force while at rest, this force may cause some displacement but the solid does not continue to move indefinitely.

Q2.

At the pipe wall the velocity of the water will be zero. The velocity will increase as we move toward the centre of the pipe. This change in velocity across the direction of flow is known as **velocity profile** as shown in the figure below and it is **non-uniform**:
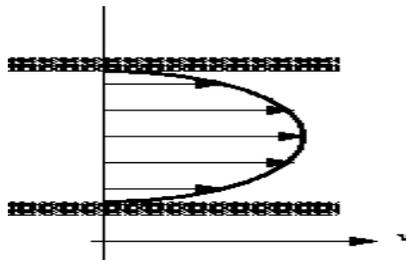
Velocity profile in a pipe.

Because particles of fluid next to each other are moving with different velocities there **are** shear forces in the moving fluid. On the other hand, if a fluid is a long way from the boundary and all the particles are travelling with the same velocity,


the velocity profile is uniform
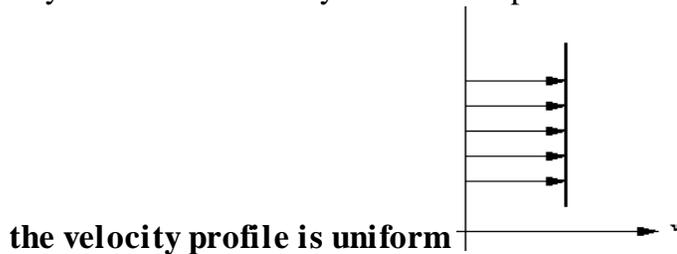
This implies all particles have zero relative velocity. In practice we are concerned with flow past solid boundaries; aeroplanes, cars, pipe walls, river channels etc. and shear forces **will** be present.

Q3.

There are several categories based on the relationship between shear stress and the velocity gradient (rate of shear strain) in the fluid. The several categories are:

Plastic, Bingham plastic, Pseudo-plastic,   Dilatant substances, Thixotropic substances, Rheopectic substances,   and Viscoelastic materials
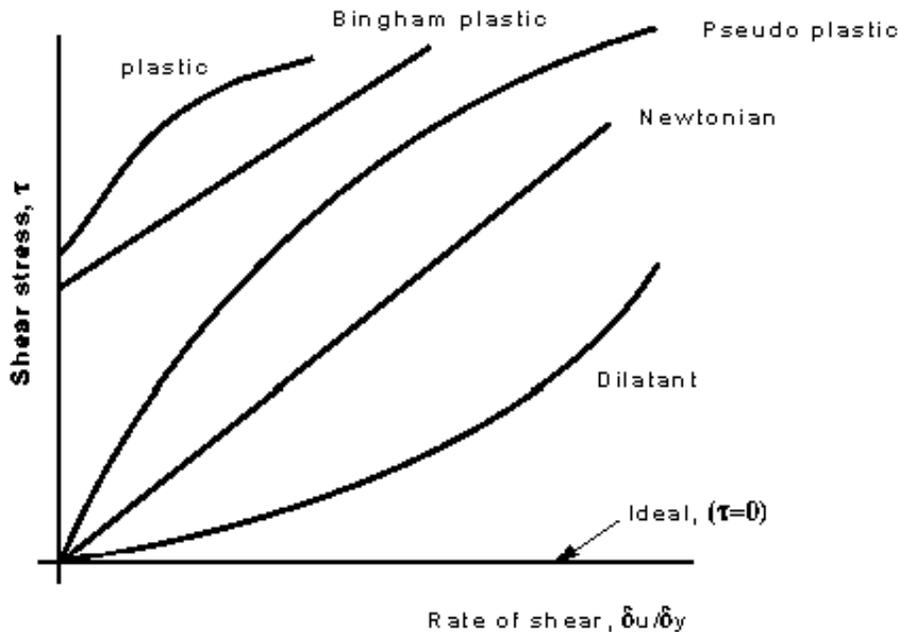
Shear stress vs. Rate of shear strain
Q4

Newtonian / Non-Newtonian Fluids

Fluids vary in behaviour under stress. Fluids obeying Newton's law where the value of coeff. of dynamic viscosity (μ is constant are known as **Newtonian** fluids. If μ is constant the shear stress is linearly dependent on velocity gradient

Q5. For Newtonian fluids A = 0, B = μ and n = 1.

Q6. Calculate the viscosity for each section of the line

du/dy (s^{-1})	0.0	0.3	0.5	0.7	0.9
τ (N m^{-2})	0.0	1.5	2.4	3.6	4.4
Viscosity	0.0	5.0	1.6	5.1	4.9

Thus the mean viscosity = 3.32 N s / m^2

6

Q7.

$$\mu = \tau \, / \frac{du}{dy}$$

$$\tau = \frac{force}{area} = \frac{massxacc \; \ln}{area} = MLT^{-2}L^{-2}$$

$$\frac{du}{dy} = \frac{velocity}{dis \tan ce} = LT^{-1}L^{-1}$$

$$\mu = \frac{MLT^{-2.}L^{-2}}{LT^{-1}L^{-1}}$$

$$= MT^{-1}L^{-1}$$

Q8.
Distinct Characteristics of liquid and gases

Although liquids and gases behave much in the same way and share many similar characteristics, they also possess distinct characteristics of their own. Specifically:

- A liquid is difficult to compress and often regarded as being incompressible.
- A gas is easily compressed and usually treated as such - it changes volume with pressure.
- A given mass of liquid occupies a given volume and will occupy the container it is in and form a free surface on top(if the container is of a larger volume).
- A gas has no fixed volume, it changes volume and expand to fill the containing vessel. It will completely fill the vessel so no free surface is formed.

Q9.

Viscosity in Liquids

There is some molecular interchange between adjacent layers in liquids - but as the molecules are so much closer than in gases the cohesive forces hold the molecules in

place much more rigidly. This cohesion plays an important roll in the viscosity of liquids. Increasing the temperature of a liquid reduces the cohesive forces and increases the molecular interchange. Reducing cohesive forces reduces shear stress, while increasing molecular interchange increases shear stress.

Viscosity in Gases

The molecules of gases are only weakly kept in position by molecular cohesion (as they are so far apart). As adjacent layers move by each other there is a continuous exchange of molecules. Molecules of a slower layer move to faster layers causing a drag, while molecules moving the other way exert an acceleration force. Mathematically this momentum exchange can lead to Newton law of viscosity. If temperature of a gas increases the momentum exchange between layers will increase thus increasing viscosity. Viscosity will also change with pressure - but under normal conditions this change is negligible in gases.

Q10.

Mass Density: Mass Density, ρ , is defined as the mass of substance per unit volume.

Units: kgm^{-3}

Dimensions: ML^{-3}

Specific Weight: Specific Weight ω, (and sometimes known as *specific gravity*) is defined as the weight per unit volume.

or The force exerted by gravity, g, upon a unit volume of the substance.

The Relationship between g and ω can be determined by Newton's 2nd Law, weight per

unit volume $= \dfrac{m \cdot g}{v} = \dfrac{m}{v} \cdot g$,

$$\omega = \rho g$$

Units: Nm^{-3}

Dimensions: $ML^{-2}T^{-2}$.

Relative Density, ⌐F , is defined as the ratio of mass density of a substance to some standard mass density.

Units: None, since a ratio is a pure number.

Dimensions: 1.

Q11

ρ_{oil} = **850 kg/m³** ρ water = **1000 kg/m³**

γ oil = **850 / 1000 = 0.85**

Dynamic viscosity = μ = 5 x 10⁻³ kg/ms

Kinematic viscosity $\nu = \mu / \rho$

$$\nu = \frac{\mu}{\rho} = \frac{5 \times 10^{-3}}{1000} = 5 \times 10^{-6} \, m^2 s^{-1}$$

Q 12.

$$u = 0.68y - y^2$$
$$\frac{\partial u}{\partial y} = 0.68 - 2y$$

At the plate face y = 0m,

$$\frac{\partial u}{\partial y} = 0.68$$

The shear stress at the plate face is:

$$\tau = \mu\frac{\partial u}{\partial y} = 0.9 \times 0.68 = 0.612\, N/m^2$$

At y = 0.34m,

$$\frac{\partial u}{\partial y} = 0.68 - 2 \times 0.34 = 0.0$$

As the velocity gradient is zero at y = 0.34 then the shear stress must also be zero

Q13.

A static fluid has following properties

- has **no shearing force** acting on it,

- any force of interaction between the fluid and the boundary acts perpendicular to the boundary.

- the sum of the components of forces in any direction is zero.

The sum of the moments of forces on the element about
Q14.

Consider a small element of fluid in the form of a triangular prism which contains a point P, establish a relationship between the three pressures p_x in x-direction, p_y in y-direction and p_s in direction normal to the sloping face
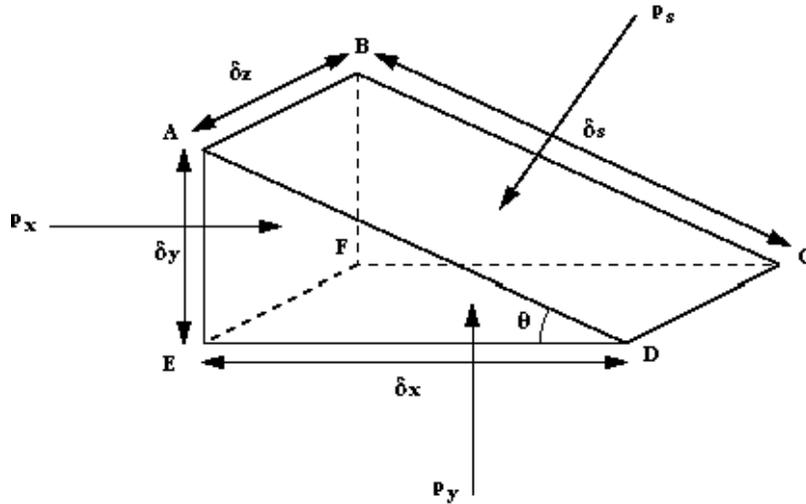
The fluid is at rest, so we know there are no shearing forces, and we know that all force are acting at right angles to the surfaces .i.e.

p_s acts perpendicular to surface ABCD,

p_x acts perpendicular to surface ABFE and

p_y acts perpendicular to surface FECD.

And, as the fluid is at rest, i.e in equilibrium, the sum of the forces in any direction is zero.

Summing forces in the x-direction:

Force due to p_x,

$$F_x = p_x \times Area_{ABFE} = p_x \partial z \partial y$$

Component of force in the x-direction due to p_s,

$$F_{x_s} = -p_s \times Area_{ABCD} \times \sin \theta$$

$$= -p_s \delta s \, \delta z \frac{\delta y}{\delta s}$$

$$= -p_s \delta y \, \delta z$$

$$(\quad \sin \theta = \frac{\delta y}{\delta s} \quad)$$

Component of force in x-direction due to p_y,

$$F_{x_y} = 0$$

To be at rest (i.e. in equilibrium)

$$F_{xx} + F_{xs} + F_{xy} = 0$$

$$p_x \partial z \partial y - p_s \partial y \partial z = 0$$

$$p_x = p_s$$

Similarly, summing forces in the y-direction. Force due to p_y,

$$F_{y_y} = p_y \times Area_{BFCD} = p_y \delta x \delta z$$

Component of force due to p_s,

$$F_{y_s} = -p_s \times Area_{ABCD} \times \cos\theta$$

$$= -p_s \delta s \delta z \frac{\delta x}{\delta s}$$

$$= -p_s \delta x \delta z$$

$$\cos\theta = \frac{\delta x}{\delta s}$$

Component of force due to p_x,

$$F_{y_x} = 0$$

Force due to gravity,

$$weight = -specific\ weight \times volume\ of\ element$$

$$= -\rho g \times \frac{1}{2} \delta x \delta y \delta z$$

To be at rest (in equilibrium)

$$F_{y_y} + F_{y_s} + F_{y_x} + \text{weight} = 0$$

$$p_y \delta x \delta y + \left(- p_s \delta x \delta z\right) + \left(- \rho g \frac{1}{2} \delta x \delta y \delta z\right) = 0$$

The element is small i.e. δx, δy and δz are small, and so $\delta x \delta y \delta z$ is <u>very</u> small and considered negligible, hence

$$p_y = p_s$$

thus
$$p_x = p_y = p_s$$

The derived expression $p_x = p_y = p_s$. indicates that pressure at any point is the same in all directions. This is known as **Pascal's Law** and applies to fluids at rest.

Q15.

$$Q_1 = A_1 u_1 = \left(\frac{\pi d^2}{4}\right) u$$

$$= 0.00392 \, m^3 / s$$

$$Q_2 = 0.3 Q_1 = 0.001178 m^3 / s$$

$$Q_1 = Q_2 + Q_3$$

$$Q_3 = Q_1 - 0.3 Q_1 = 0.7 Q_1$$

$$= 0.00275 m^3 / s$$

$$Q_2 = A_2 u_2$$
$$u_2 = 0.936 m/s$$
$$Q_3 = A_3 u_3$$
$$u_3 = 0.972 m/s$$

Q16.

Flow is *two-dimensional* if it can be assumed that the flow parameters vary in the direction of flow and in another direction at right angles to this direction. Streamlines in two-dimensional flow are curved lines on a plane and are the same on all parallel planes. An example is flow over a weir foe which typical streamlines can be seen in the figure below (irrigation dam). Over the majority of the length of the weir the flow is the same - only at the two ends does it change slightly. Here correction factors may be applied.
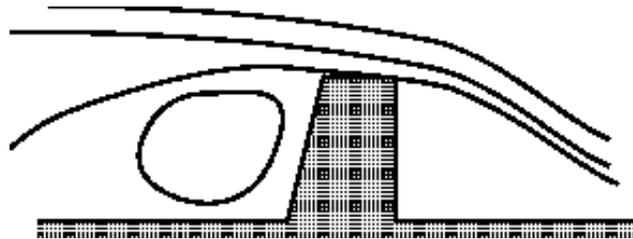
Two-dimensional flow over a weir.

Q17.

The derivation of Bernoulli's Equation:

15
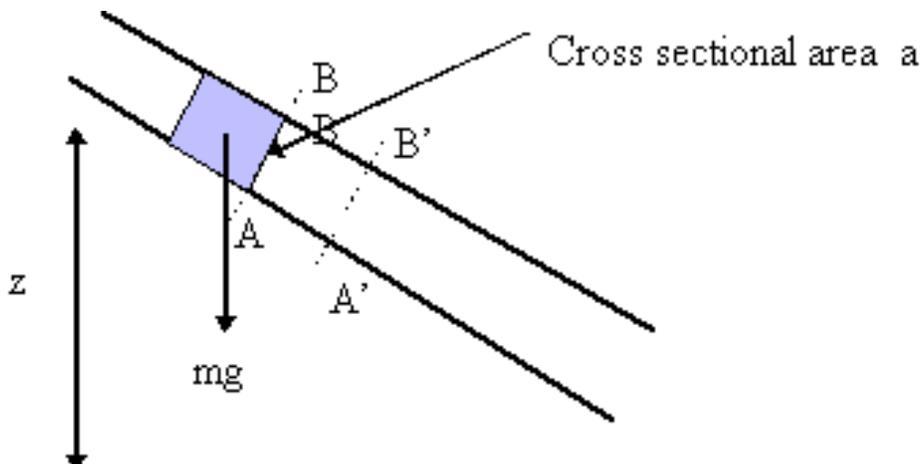
An element of fluid, as that in the figure above, has potential energy due to its height z above a datum and kinetic energy due to its velocity u. If the element has weight mg then

potential energy = mgz

potential energy per unit weight $= mgz/mg = z$

kinetic energy $= \frac{1}{2}mu^2$

kinetic energy per unit weight $= mu^2/2mg = u^2/2g$

At any cross-section where the pressure generates a force, the fluid will flow, moving the cross-section, so work will be done. If the pressure at cross section AB is p and the area of the cross-section is a , then

force on AB = pa

as the weight mg of fluid passed AB, cross-section AB moved to A'B'

volume passing AB $= \dfrac{mg}{\rho g} = \dfrac{m}{\rho}$

So, distance AA' $= \dfrac{m}{\rho a}$

work done = force x distance AA'

$$= \quad pa \times \dfrac{m}{\rho a} = \dfrac{pm}{\rho}$$

work done per unit weight =

$$\dfrac{p}{\rho g}$$

This term is known as the <u>pressure energy</u> of the flowing stream.

Summing all of these energy terms gives

$$
\begin{array}{cccc}
\text{Pressure} & \text{Kinetic} & \text{Potential} & \text{Total} \\
\text{energy per} + & \text{energy per} + & \text{energy per} = & \text{energy per} \\
\text{unit weight} & \text{unit weight} & \text{unit weight} & \text{unit weight}
\end{array}
$$

or

$$
\frac{p}{\rho g} + \frac{u^2}{2g} + z = H
$$

As all of these elements of the equation have units of length, they are often referred to as the following:

pressure head = p/ρg

velocity head = u^2/2g

potential head =z

total head = *H*

By the principle of conservation of energy the total *energy* in the system does not change, Thus the total *head* does not change. So the Bernoulli equation can be written

$$
\frac{p}{\rho g} + \frac{u^2}{2g} + z = H = \text{Constant}
$$

As stated above, the Bernoulli equation applies to conditions along a streamline. We can apply it between two points, 1 and 2, on the streamline in the figure below
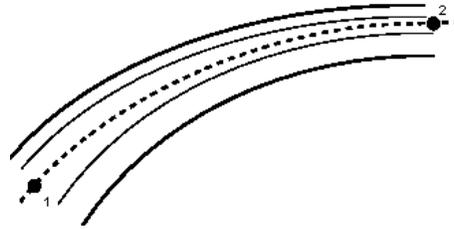
Two points joined by a streamline

total energy per unit weight at 1 = total energy per unit weight at 2

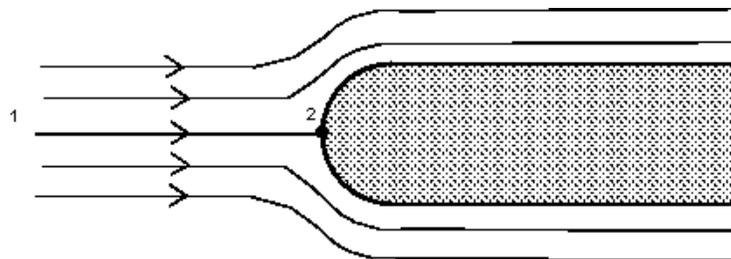or total head at 1 = total head at 2   or

This equation assumes no energy losses (e.g. from friction) or energy gains (e.g. from a pump) along the streamline.

Q18.

Pitot Tube

If a stream of uniform velocity flows into a blunt body, the stream lines take a pattern similar to this:

Streamlines around a blunt body

The fluid stops because at point2 because the velocity is zero - the fluid does not move at this one point. This point is known as the *stagnation point.*

From the Bernoulli equation we can calculate the pressure at this point. Apply Bernoulli along the central streamline from a point upstream where the velocity is u_1 and the pressure p_1 to the stagnation point of the blunt body where the velocity is zero, $u_2 = 0$. Also $z_1 = z_2$.

18

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

$$\frac{p_1}{\rho} + \frac{u_1^2}{2} = \frac{p_2}{\rho}$$

$$p_2 = p_1 + \frac{1}{2}\rho u_1^2$$

This increase in pressure which bring the fluid to rest is called the *dynamic pressure*.

Q 19.

At stagnation point the velocity is u_2 and the pressure is p_2, we have

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

In the pitot tube, height $z_1 = z_2$, and velocity at stagnation point $u_2 = 0$

$$\frac{p_1}{\rho} + \frac{u_1^2}{2} = \frac{p_2}{\rho}$$

Therefore,

$$\frac{u_1^2}{2} = \frac{p_2 - p_1}{\rho} = \frac{1710 - 1250}{850}$$

$$\frac{450}{850} = 0.54$$

$$\Rightarrow u_1^2 = 2 \times 0.54 = 1.08$$

$$u_1 = 1.04 \text{m/s}$$

Stagnation Head

$$h_2 = p_2/\rho g = 1710/(850 \times 9.8)$$
$$= 0.205 \text{m} = 205 \text{mm}$$

Q 20.

$$\rho_g g = 19.62 \ N/m^2$$
$$C_d = 0.96$$
$$d_1 = 0.3m$$
$$d_2 = 0.2m$$

Calculate Q.

$$u_1 = Q/0.0707 \qquad\qquad u_2 = Q/0.0314$$

For the manometer:

$$p_1 + \rho_g gz = p_2 + \rho_g g\left(z_2 - R_p\right) + \rho_w g R_p$$
$$p_1 - p_2 = 19.62\left(z_2 - z_1\right) + 587.423 \qquad\qquad <----(1)$$

For the Venturimeter

$$\frac{p_1}{\rho_g g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho_g g} + \frac{u_2^2}{2g} + z_2$$

$$p_1 - p_2 = 19.62\left(z_2 - z_1\right) + 0.803 u_2^2 \qquad \leftarrow ---- (2)$$

Combining (1) and (2)

$$0.803 u_2^2 = 587.423$$

$$u_{2_{ideal}} = 27.047 \, m/s$$

$$Q_{ideal} = 27.047 \times \pi \left(\frac{0.2}{2}\right)^2 = 0.85 m^3/s$$

$$Q = C_d Q_{idea} = 0.96 \times 0.85 = 0.816 m^3/s$$