COVENANT UNIVERSITY NIGERIA

TUTORIAL KIT
OMEGA SEMESTER

PROGRAMME: PHYSICS

COURSE: PHY 225

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PHY 225: Mathematical Methods in Physics (I)

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1. Show that the set of vectors ?, ?, ? are linearly independent. If

$$? = ? + 2 ? + 2 ?$$
$$? = ? + ? + ?$$
$$? = ? + 2 ? + ?$$

Solution

If the set of vectors ?, ?, ? are linearly independent, then it must satisfy the condition

$$? = ? + 2 ? + 2 ?$$
$$? = ? + ? + ?$$
$$? = ? + 2 ? + ?$$

$$? = ? ? + ? ? + \ldots\ldots\ldots + ? ? = 0$$

$$? ? + 2 ? + ? + ? ? + ? + ? + ? ? + 2 ? + ? = 0$$

$$?? ? ? + ?? ? ? 2 ? + ? - 2 ? ? ? ? = 0$$

Hence,

$$? ? ? = 0$$
$$? ? ? 2 ? = 0$$
$$- 2 ? ? ? ? = 0$$

Solving the sets of equations give

$$? = 3$$
$$? = 5$$
$$? = ?$$

Hence, the condition $? = ? ? + ? ? + \ldots\ldots\ldots + ? ? = 0$ is satisfied. Therefore vectors ?, ?, ? are linearly independent.

2. Two particles emitting from a source have displacement at any time. Find the relative displacement of the second particle with respect to the first.

$$? = 8 ? + 6 ? + 16 ?$$
$$? = 4 ? + 10 ? + 10 ?$$

3. $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$, $\vec{B} = 3\hat{i} + 4\hat{j} + \hat{k}$. What unit vector is perpendicular to both $\vec{A}$ and $\vec{B}$? Find the angle between these vector

Solution

The unit vector is perpendicular to both $\vec{A}$ and $\vec{B}$ is the unit vector in the direction of $\vec{A}\times\vec{B}$

$$\vec{A}\times\vec{B} = (8\hat{i} + 6\hat{j} + 16\hat{k}) \times (4\hat{i} + 10\hat{j} + 10\hat{k})$$

$$= -3\hat{i} + 5\hat{j} + 11\hat{k}$$

$$|\vec{A}\times\vec{B}| = \sqrt{155}$$

Therefore, the required unit vector $\hat{n}$ is

$$\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$$

$$|\vec{A}| = \sqrt{6}$$

$$|\vec{B}| = \sqrt{26}$$

$$|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta$$

$$|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta$$

$$\sin\theta = \frac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|} = \frac{\sqrt{155}}{\sqrt{26\times 6}}$$

4. The electric field $\vec{E}$, magnetic field $\vec{B}$ and velocity $\vec{v}$ of a charged particle q are given by

$$\vec{E} = 4\hat{i}$$

$$\vec{B} = 9\hat{i} + 12\hat{j}$$

$$\vec{v} = 0\hat{i} + 3\hat{j} + 4\hat{k}$$

Determine the electrostatic and magnetic force on the charged particle

5. Show that the line of intersection of the planes x+2y+3z=0 and 3x+2y+z=0 is equally inclined to the x and z axes and makes an angle [illegible] with the y-axis.

A line is given by $\vec{r} = \vec{a} + \lambda\vec{b}$, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 4\hat{i} + 5\hat{j} + 6\hat{k}$. Find the coordinates of the point P at which the line intersects the plane x+2y+3z=6

Solution

The vector normal to the plane is $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$ hence,

$$\vec{b}\cdot\vec{n} = (4\hat{i} + 5\hat{j} + 6\hat{k})\cdot(\hat{i} + 2\hat{j} + 3\hat{k}) = 32 \neq 0$$

This means that the line indeed intersect the plane

$\vec{r} = ? + \lambda ? = (1 + 4\lambda)\hat{i} + (2 + 5\lambda)\hat{j} + (3 + 6\lambda)\hat{k}$

To find the point of intersection, we substitute the above equation into the equation x+2y+3z=6, so that $(1 + 4\lambda) + 2(2 + 5\lambda) + 3(3 + 6\lambda) = 6$

$$\lambda = -\frac{1}{4}$$

$$x = 1 + 4\lambda = 0$$

$$y = 2 + 5\lambda = \frac{3}{4}$$

$$z = 3 + 6\lambda = \frac{3}{2}$$

Hence the point of intersection is (0, [illegible])

6. A particle of mass 2Kg moving with initial velocity of (2 ? + 4 ?) m/s is acted upon by a constant force (2 ? + 4 ? − 4 ?) N. Determine the distance and the velocity after 5s. Find the time in which the particle reaches the xy-plane.

7. Show that ? satisfies the partial differential equation

$$[illegible] \; 2 \; [illegible] \; 1 \; [illegible] = 0$$

8. Find the gradient of the scalar field Ø = 3 ?

Solution

$$Ø = 3 ?$$

$$\nabla Ø = ? \; ? + ? + ? \; ?3 ?$$

$$= ? \; ?3 ? + ? ? + ? \; ?3 ?$$

$$= 3 ? \; 6 ? \; 9 ?$$

9. Find the rate of change with respect to distance S of Ø = 3 ? at the point (1, 2, -1) in the direction ? + 2 ? + 3 ?. In which direction is the rate of change of Ø greatest at this point. What is the value in this direction?

10. Find the divergence of the vector field ?

Solution

? . ? ? + ? + ? . ?

= ? + ? + ?

= 2 ? 2 ? 2 ?

11. Find the laplacian of the scalar field Ø =

12. Find the curl of the function ?

Solution

? × ? ? + ? + ? × ? ?

= ? ?

= 0 +

13. Express the vector ? = ? in spherical polar coordinates. Find ? × ? and ? . ?

14. Express the vector ? = ? in cylindrical polar coordinates. Find ? . ?

Solution

? = ?

? = - ?

By solving for ? and ? we have

? = ?

? = - ?

Hence, ? = ? - ?

Applying same protocol for ? and ?

Thus

? = ? - ?

? = ? + ?

By substituting ? , ? into ? = ... ,then

? = (? - ?) - (? + ?) + ...

Hence

$$?.? = \frac{1}{?} ... \frac{1}{?} ...$$

$$?.? = 2 ...$$

15. Evaluate the integral ... , where c is the circle in the xy plane defined by ... , z=0
16. Given that ?(?) = 2 ? ... when t=2 and ?(?) = 4 ? 2 ? + 3 ? when t=3. Show that ... = 10

Solution

$$? ? \frac{1}{2} ? ... \frac{1}{2} ...$$

$$= \frac{1}{2} ...$$

$$= \frac{1}{2}[?4 ? 2 ? + 3 ? ... + ?2 ? ? + ? ?]$$

$$= \frac{1}{2}[?16 + 4 + 9? + ?4 + 1 + 4?]$$

$$= \frac{1}{2}[?29 - 9? = 10$$

17. A vector field ? is given as ? = 4 ... , V is the region bounded by the surface x=0, y=0, y=6, $Z=x^2$, z=4. Evaluate the integral ? ...
18. State the Stoke's theorem

Solution

Stoke's theorem is the curl analogue of the divergence theorem. It relates the integral of the curl of a vector field over an open surface S to the line integral of the vector field around the perimeter C bounding the surface

19. Proof mathematically that Gauss law is valid for any Gaussian surface

Solution

Assume a cube. The divergence theorem is

we have

$$= 2$$

Now the volume integral

$$? \quad ? \quad = ? \quad 2 * ?x + y + z?dxdydz$$

$$? \quad ? \quad ? \quad 2 * ?x + y + z?dxdydz$$

$$= 2 ?\frac{1}{2} + \frac{1}{2} + \frac{1}{2}? = 3$$

The surface integral is described by the six faces of the cube

$$? \quad = ? \quad ?$$

$$+ ? \quad ? \quad ? \quad ?$$

$$? \quad ?$$

$$? \quad ? \quad 0 \quad 0$$

$$? \quad ?$$

$$= ? \quad ? \quad ? \quad = 1$$

$$? \quad ? \quad 0 \quad 0$$

$$? \quad ?$$

$$= ? \quad ? \quad ? \quad = 1$$

[illegible]

[illegible] $= 0 + 1 + 0 + 1 + 0 + 1 = 3$

It follows that the volume integral is equal to the surface integral

20. Compute [illegible], where [illegible] and C is the curve of intersection of the plane [illegible] and the cylinder [illegible] 4, oriented in the counterclockwise direction when viewed from above