

COVENANT UNIVERSITY  
NIGERIA

*TUTORIAL KIT*  
*OMEGA SEMESTER*

PROGRAMME: PHYSICS

COURSE: PHY 421

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# PHY 421: Control Systems

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- (1) Define a control system.
- (2) Give four examples of control system devices.
- (3) Define the following: Controlled Variable and manipulated variable, Plant,
- (4) Explain the following: Disturbances, and Feedback Control
- (5) State four Advantages of control systems
- (6) Consider the electric network shown in Figure 1, without using any equations, discuss how many states the system has and build a state space model.

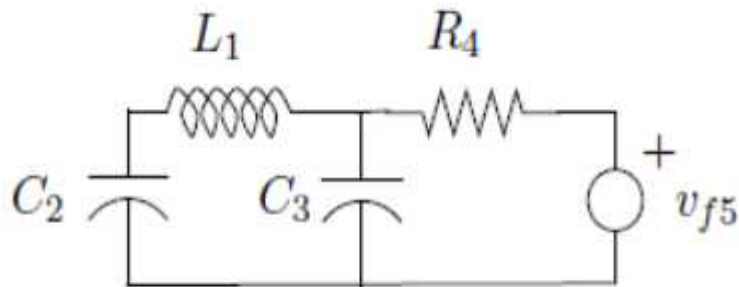


Figure 1: Electric network

- (7) With the aid of block diagrams explain open-loop and closed-loop control systems and explain the difference between the two systems.
- (8) List the six established orderly sequence for the design of feedback control systems
- (9) Consider a water level control system of a storage tank of cross-sectional area  $A$  whose liquid level or height is  $h$  shown in Figure 4. The liquid enters the tank from the top and leaves the tank at the bottom through the valve, whose fluid resistance is  $R$ . The volume flow rate in and the volume flow rate out are  $q_i$  and  $q_o$ , respectively. The fluid density is constant. In such a system it is desired to regulate the water level in the tank. Assume that the variable that we can change to control the water level is  $q_i$ .

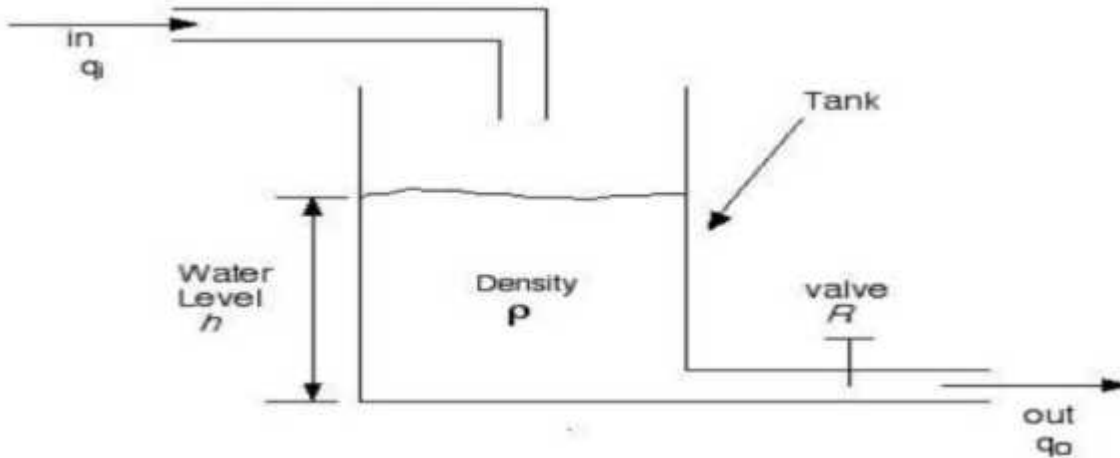


Figure 4: A water level control system of a storage tank

- (i) Identify the input and output of the system
- (ii) Obtain the differential equation of the system

(10) Control systems must be designed to be stable, explain the stability criteria of a system.

(11) Figure 3 below shows the transient response curve of a practical control system.

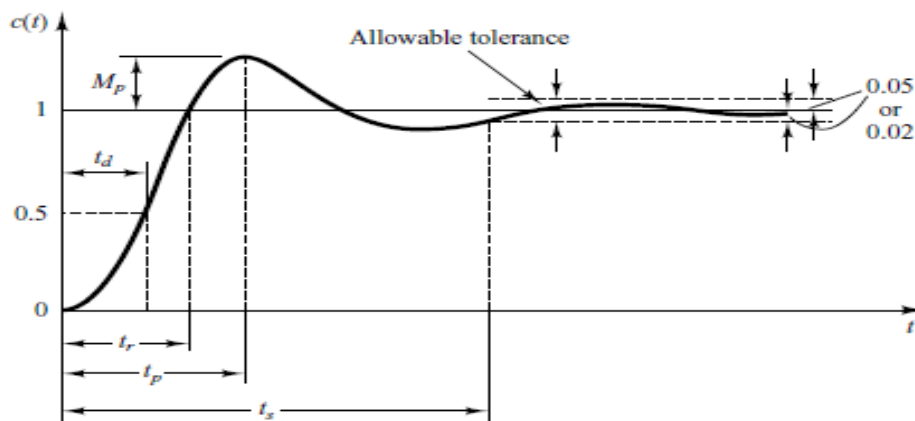


Figure 3: A unit-step response curve

Define the following

- i. Delay time, ***t<sub>d</sub>***, ii. Rise time, ***t<sub>r</sub>***
- iii. Peak time, ***t<sub>p</sub>***, iv. Maximum overshoot, ***M<sub>p</sub>***, v. Settling time, ***t<sub>s</sub>***

(12) Write a Matlab program to obtain the Unit-Step response of the Transfer-function of a system given by:

$$G(s) = \frac{25}{s^2 + 4s + 25}$$

(13) List three major objectives of system analysis and design.

(14) Consider a single tank of constant cross-sectional area  $A$ . The flow of water from the tank is governed by the relationship

$$f_{out} = K\sqrt{h}$$

where  $h$  is the height of liquid in the tank and  $K$  is a constant. Assume that the flow of liquid into the tank is a control variable,  $u$ .

- i.) Write down the equation governing the height of liquid in the tank.
- ii.) Linearize the model about a nominal height of  $h = h^*$ .
- ii.) Repeat part (i) and (ii) for a tank where the cross sectional area increases with height i.e.,  $A = ch$ .

(15) List four Important Design Considerations in control systems

(16) Write a Matlab code to solve the following transfer function of a control system and hence write down the final solution.

$$\frac{B(s)}{A(s)} = \frac{2s^3 + 5s^2 + 3s + 6}{s^3 + 6s^2 + 11s + 6}$$

(17) Write a Matlab code to obtain the zeros, poles and gain  $K$  of the following transfer function of a control system hence write down the zeros, poles and gain  $K$  of the system.

$$\frac{B(s)}{A(s)} = \frac{4s^2 + 16s + 12}{s^4 + 12s^3 + 44s^2 + 48s}$$

(18) Consider the simple electrical network shown in Figure 2 below. Obtain the state space model of the output voltage  $v(t)$  by applying the fundamental network laws.

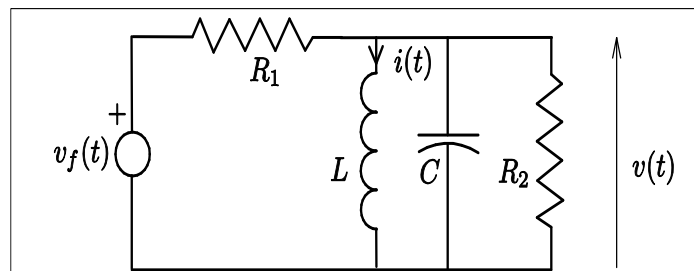


Figure 2: Electrical network. State space model

(19) Consider the following transfer function of a control system write a Matlab code to plot the Bode diagram and title the graph.

$$G(s) = \frac{25}{s^2 + 4s + 25}$$

(20) Consider the following open-loop transfer function. Write a Matlab code to draw a Nyquist plot of the system and title the graph.

$$G(s) = \frac{1}{s^2 + 0.8s + 1}$$

SOLUTION

(1) A control system consists of subsystem and processes (or plants) assembled for the purpose of obtaining a desired output with desired performance, given a specified input.

(3) Define the following:

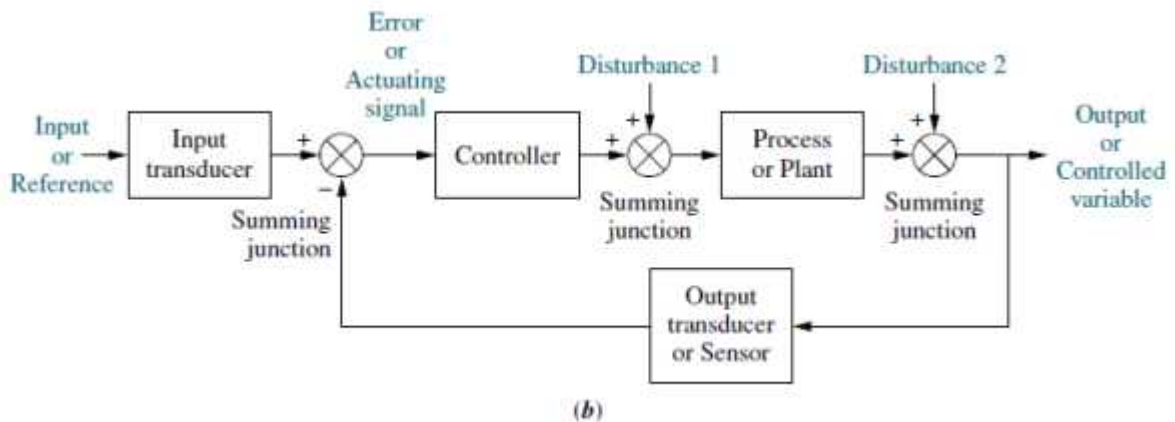
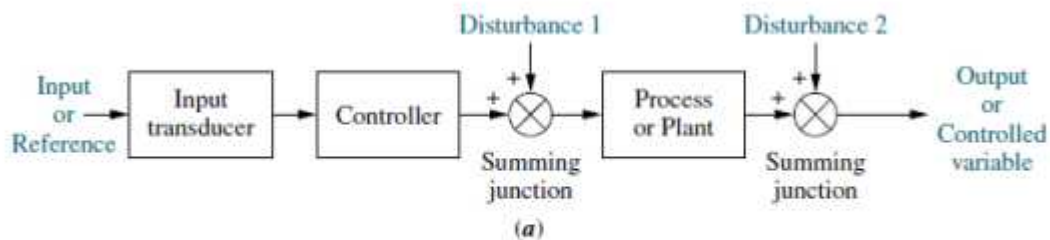
The controlled variable is the quantity or condition that is measured and controlled. The manipulated variable is the quantity or condition that is varied by the controller so as to affect the value of the controlled variable.

Plants. A plant may be a piece of equipment, perhaps just a set of machine parts functioning together, the purpose of which is to perform a particular operation

(5) Advantages of control systems

- Power amplification
- Remote control
- Convenience of input form
- Compensation for disturbances.

(7) The block diagram is shown below



Block diagrams of control systems: (a) open-loop system; (b) closed-loop system

The difference between the two systems

- i. The distinguishing characteristic of an open-loop system is that it cannot compensate for any disturbances that add to the controller's driving signal.
- ii. The output of an open-loop system is corrupted not only by signals that add to the controller's commands but also by disturbance at the output the system cannot correct for these disturbances, either.

- iii. Closed-loop systems are more complex and expensive than open-loop systems. A standard, open-loop toaster serves as an example: it is simple and inexpensive.
- iv. The closed-loop system compensates for disturbances by measuring the output response, feeding that measurement back through a feedback path, and comparing that response to the input at the summing junction. If there is any difference between the two responses, the system drives the plant, via the actuating signal, to make a correction.
- v. In summary, systems that perform the previous described measurement and correction are called closed-loop, or feedback control, systems. Systems that do not have this property of measurement and correction are called open-loop systems.

(9) (i) The input and output of the system are

Input: Volume flow rate in,  $q_i$ ,

output: Water level,  $h$ .

(ii) In order to obtain the differential equation of the system we use the conservation of mass principle which states that;

The time rate of change of fluid mass inside the tank = the mass flow rate in - mass flow rate out

$$\frac{d}{dt}(\rho A h) = \rho q_i - \rho q_o$$

$$\rho A \frac{dh}{dt} = \rho q_i - \frac{\rho g h}{R}$$

$$A \frac{dh}{dt} + \frac{g}{R} h = q_i \dots (1)$$

where,  $A$  is the cross sectional area of the tank,  $g$  is the acceleration due gravity and  $R$  is the fluid resistance through the valve. Note that the input and output appear in this differential equation.

**(11) 1. Delay time,  $t_d$ :** The delay time is the time required for the response to reach half the final value the very first time.

**2. Rise time,  $t_r$ :** The rise time is the time required for the response to rise from 10% to 90%, 5% to 95%, or 0% to 100% of its final value. For underdamped second order systems, the 0% to 100% rise time is normally used. For overdamped systems, the 10% to 90% rise time is commonly used.

**3. Peak time,  $t_p$ :** The peak time is the time required for the response to reach the first peak of the overshoot.

**4. Maximum (percent) overshoot,  $M_p$ :** The maximum overshoot is the maximum peak value of the response curve measured from unity. If the final steady-state value of the response differs from unity, then it is common to use the maximum percent overshoot. It is defined by

$$\text{Maximum percent overshoot} = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100\%$$

The amount of the maximum (percent) overshoot directly indicates the relative stability of the system.

**5. Settling time,  $t_s$ :** The settling time is the time required for the response curve to reach and stay within a range about the final value of size specified by absolute percentage of the final value (usually 2% or

5%). The settling time is related to the largest time constant of the control system. Which percentage error criterion to use may be determined from the objectives of the system design in question.

(13) Three major objectives of system analysis and design

- i. Producing the desired transient response
- ii. reducing steady-state error and
- iii. achieving stability

(15) Four Important Design Considerations in control systems

- i. Hardware selection, such as motor sizing to fulfil power requirement and
- ii. Choice of sensors for accuracy must be considered early in the design.
- iii. Finances, control system designer cannot create designs without considering their economic impact. Such considerations as budget allocations and competitive pricing must guide the engineer.
- iv. Robust design: systems parameter considered constant during the design for transient response, steady-state errors, and stability change over time when the actual system is built. Thus the engineer must create a robust design so that the system will not be sensitive to parameter changes.

(17) The Matlab code to obtain the zeros, poles and gain K is

```
num = [0 0 4 16 12];  
den = [1 12 44 48 0];  
[z, p, k] = tf2zp(num, den)
```

The computer will produce the following output on the screen

```
z = -3, -1  
p = 0, -6.0000, -4.0000, -2.0000  
k = 4
```

The interpretation of the solution are:

The zero at  $s = -3$  and  $-1$   
The poles at  $s = 0, -6, -4,$  and  $-2$ .  
The gain  $k$  is  $4$ .

(19) The Matlab code to plot the Bode diagram and title the graph is

```
num = [0 0 25];  
den = [14 25];  
bode(num, den)  
title('Bode Diagram of  $G(s) = 25/(s^2 + 4s + 25)$ ')
```