

Optimal advert placement slot – using the knapsack problem model

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ABSTRACT

The Knapsack problem model is a general resource allocation model in which a single resource is assigned to a number of alternatives with the objective of maximizing the total return. In this work, we applied the knapsack problem model to the placement of advert slots in the media. The aim was to optimize the capital allocated for advert placements. The general practice is that funds are allocated by trial and error and at the discretions of persons. This approach most times do not yield maximum results, lesser audience are reached. But when the scientific Knapsack problem model was applied to industry data, a better result was achieved, wider audience and minimal cost was attained.

Keywords: Knapsack model, advert placement, advertising, resource allocation.

INTRODUCTION

The Knapsack model is a general resource allocation model in which a single resource is assigned to a number of alternatives with the objective of maximizing the total return. The model is one of the most important classes of applications of Dynamic programming. The model is a distribution of effort problem that has a linear objective function and a single linear constraint. This model is also know as the fly-away kit problem or the cargo-loading problem, Taha (2008).

Assume you have a bag which can contain a certain weight, say w . There are various items you want to carry in the bag, but the total weight of these items is greater than the weight the bag can carry. If each item is given a value, say v_i and has a weight w_i for each i (such that $i = 1, 2, 3, \dots, N$, where N is the total number of items) and m_i the number of units of item i in the bag; this model determines the most valuable items to be carried in the bag and in what quantity. That is, it determines what items, considering individual weight and value that should be carried so as to have almost all that is needed such that those carried are 'worth more' than those left behind Smith (1991). In a sense, the most important factors to identify in a Knapsack problem model are the weight the bag can carry, the weight of each items to be considered and the value of each of these items.

This work applies the Knapsack model to the advertising industry. The aim is to optimize the capital allocated for advertising in any business so that the business gets the best combination of adverts,

through different media that would reach the largest audience possible with a minimal cost.

The general practice is that most establishments do not have a well structured plan on how to allocate funds for advertising. Funds are allocated by trial and error and at the discretion of persons or departments in charge. These methods are faulted, and are basically inefficient as funds available are not optimally utilized. In this work, we propose that to get the best combination of adverts through different advertising agent that would reach the largest audience possible for a minimal cost, the Knapsack model should be adopted.

The general problem is represented by the following:

$$\text{Maximize } Z = v_1 m_1 + v_2 m_2 + \dots + v_n m_n$$

$$\text{subject to } w_1 m_1 + w_2 m_2 + \dots + w_n m_n \leq w$$

$$m_i \in Z^+ \quad (1)$$

The Knapsack problem model has been applied to many real life applications either a stand alone model or as a combination of models. Eilon and Williamson (1998) developed BARK (Budget Allocation by Ranking and Knapsack) to solve a particular problem in determining which projects should be selected, from a given array, for implementation subject to budgetary constraints. This problem is often encountered in the public sector, where the values, in financial terms, of competing projects are difficult or impossible to quantify, but where the projects may be ranked in terms of their perceived worth or benefit.

Gerard et al (2008) considered a procurement problem where suppliers offer concave quantity discounts. The resulting continuous Knapsack problem involves the minimization of a sum of separable concave functions.

Argali and Geunes (2009) considered the allocation of a limited budget to a set of activities or investments in order to maximize return from investment. In a number of practical contexts, for example, advertising, the return from investment in an activity is effectively modelled using an S-curve, where increasing returns to scale exist at small investment levels, and decreasing returns to scale occur at high investment levels.

Kalai and Vanderpooten (2006) studied the robust Knapsack problem using a max-min criterion. The Knapsack problem is a classical combinatorial problem used to model many industrial situations.

Our own contribution to the literature is that for the first time, to the best of our knowledge, the Knapsack problem model is applied to the advertising for optimal allocation of advertising slots for minimal cost.

MATERIALS AND METHODS

As mentioned before, it is important to place adverts in such a way that the combination of adverts would have the largest exposure possible. That is, the widest reach. The problem here is to select media types for adverts in such a way that the widest reach would have been achieved without over shooting the amount allocated for adverts.

In comparison to the Knapsack problem model, the holding capacity of the bag is the resource limit, given here as the advertising budget. The items to be considered are the different medium that can be used, the weight of any item is the cost of placing an advert using this medium and the value of the item is the reach of the media type.

The hypothetical company, name not discussed for security reasons, had an advertising budget of 9 million Naira (our local currency) per month, with a maximum of 5 million Naira for use on advertisements and the rest on promotions. They can choose to place adverts using the following medium: print, television, billboards and radio. According to Burnet (1994), the viability of an advertising plan depends on its reach. Table1 gives the cost, w_i of placing an advert in each media type at a peck period and the average reach, r_i of the medium.

The problem to be solved can be stated as follows:

$$\begin{aligned} \text{Maximize } R &= r_1m_1 + r_2m_2 + r_3m_3 + r_4m_4 \\ \text{Subject to } w_1m_1 + w_2m_2 + w_3m_3 + w_4m_4 &\leq W \\ m_i &\geq 0, i = 1,2,3,4 \text{ and integer} \end{aligned}$$

Where,
R = total reach

r_i = Reach of each medium /items

m_i = Number of adverts placed using each medium

w_i = Cost of placing an advert in each medium

W = Total amount available for adverts (resource limit).

That is, the problem is to maximize the total reach, sum of the reach from each medium, such that the total cost of the adverts is less than the amount available for adverts and the number of adverts placed using each medium are non-negative and an integer.

To carry out this computation, we used the excel spread sheet. Excel spread sheet is user friendly and very affordable.

$$N = 4$$

$$W = 5 \text{ million}$$

$$i = 4$$

RESULT AND DISCUSSION

Thus, the optimum as obtained in table 6 is $(m_1 * m_2 * m_3 * m_4) = (1, 2, 0, 5)$ and the optimum revenue is $f_1(5) = 125,000,000$.

This means that the company should place 1 print, 2 TV, no billboard and 5 radio adverts to obtain the optimum reach of 125,000,000.

The amount spent is given below:

$$\text{Total amount spent} = [0.8(1) + 1.3(2) + 0.5(0) + 0.6(5)] * 1,000,000 = =N=3,700,000.$$

CONCLUSION

Optimum result is achieved in table 6. That is, the company should place one print, two TV, no billboard and 5 radio advertisement slots for the month under consideration to obtain an optimum reach of 125,000,000, at a total cost of =N=3,700,000.

Currently, as at the time of this work, there is no set method for determining what media types to be used and in what quantity by the company. The medium are chosen using guess work and by the discretion of the person(s) in charge.

For the month used for our analysis, the company using their crude approach arrived at the following conclusions; place a total of one TV, two prints and two billboards adverts. Total reach achieved was estimated to be 102,000,000 at the total cost of =N=3,900,000.

Using the more scientific Knapsack problem model gives a better result. We therefore recommend that the Knapsack problem model should be adopted for advert placement and media planning.

Table1.

Item	Medium	Cost (=N=). w_i	Reach r_i
1	Print	845,950	25,000,000
2	Television	1,320,000	40,000,000
3	Billboards	469,000	6,000,000
4	Radio	629,360	10,000,000

Table2: Scaled value of table1.

Item	Cost (=N=). w_i	Reach r_i
1	0.8	25
2	1.3	40
3	0.5	6
4	0.6	10

Stage 4

Table 3: Showing the first stage of our iteration.

Stage 3:

- Number of stages = 4
- Current stage = 3
- Resource limit, $W = 5$
- $w_3 = 0.5$
- $r_3 = 6$
- 9 m_3 values are to be entered in row 5 (starting at column D)

Table 4:

The screenshot shows an Excel spreadsheet titled "excelKnapsackSTAGE4 [Compatibility Mode] - Microsoft Excel non-commercial use". The spreadsheet is divided into several sections:

- Input Data and Stage Calculations:**
 - Number of stages, N = 4
 - Res. limit, W = 5
 - Current stage = 3
 - w3 = 0.5
 - r3 = 6
 - Are m3 values correct? (Yes/Yes/Yes/Yes/Yes/Yes/Yes/Yes/Yes/Yes)
 - m3 = 0
 - r3*m3 = 0
 - w3*m3 = 0
 - f4 = 0
 - x3 = 0, 1, 2, 3, 4, 5
 - w3 = 0, 1.3, 2.6, 3.9, 5.2, 6.5
 - r3 = 6, 12, 18, 24, 30, 36, 42, 48, 54, 60
- Output Solution Summary:**
 - Stage Optimum Solution: x=1, f=10, m=1
 - Stage 4: x=0, f=0, m=0
 - Stage 3: x=0, f=0, m=0

Stage 2:

- Number of stages = 4
- Current stage = 2
- Resource limit, W = 5
- w₂ = 1.3
- r₂ = 40
- 4 m₂ values are to be entered in row 5 (starting at column D)

Table 5:

The screenshot shows an Excel spreadsheet titled "excelKnapsackSTAGE4 [Compatibility Mode] - Microsoft Excel non-commercial use". The spreadsheet is divided into several sections:

- Input Data and Stage Calculations:**
 - Number of stages, N = 4
 - Res. limit, W = 5
 - Current stage = 2
 - w2 = 1.3
 - r2 = 40
 - Are m2 values correct? (Yes/Yes/Yes/Yes)
 - m2 = 0
 - r2*m2 = 0
 - w2*m2 = 0
 - f3 = 0
 - x2 = 0, 1, 2, 3, 4, 5
 - w2 = 0, 1.3, 2.6, 3.9, 5.2, 6.5
 - r2 = 40, 80, 120, 160, 200, 240
- Output Solution Summary:**
 - Stage Optimum Solution: x=2, f=30, m=3
 - Stage 4: x=0, f=0, m=0
 - Stage 3: x=1, f=10, m=1
 - Stage 2: x=0, f=0, m=0

Stage 1:

- Number of stages = 4
- Current stage = 1
- Resource limit, W = 5
- w₁ = 0.8
- r₁ = 25
- 6 m₁ values are to be entered in row 5 (starting at column D)

Table 6: Final Iteration.

The screenshot shows an Excel spreadsheet titled "excelKnapsackSTAGE4 [Compatibility Mode] - Microsoft Excel non-commercial use". The spreadsheet is divided into several sections:

- Input Data and Stage Calculations:**
 - Number of stages, N = 4
 - Res. limit, W = 5
 - Current stage = 1
 - w1 = 0.8
 - r1 = 25
 - Warning: <<Maximum [W/w] cannot exceed 10
 - Check: Are m1 values correct? (all "yes")
 - m1 = 0
 - r1*m1 = 0
 - w1*m1 = 0
- Stage Calculations:** A table showing calculations for stages 1 through 5, with columns for x1 and various numerical values.
- Output Solution Summary:** A table showing the optimal solution for each stage, with columns for x, f, and m.

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