A New Solution Methodology to the Material Balance Equation, for Saturated Reservoirs

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Abstract

The material balance equation (MBE) is a versatile analytical tool in petroleum reservoir engineering. Solution to the MBE is put to a predictive use for predicting reservoir performance, i.e. cumulative oil production, \( N_p \) as a function of the declining average reservoir pressure, \( \bar{P} \). For under-saturated reservoir (single-phase flow), the MBE presents \( N_p \) as an explicit function of pressure drawdown \( (\bar{P} - P_{wf}) \); hence a direct solution of the MBE yields the average pressure, \( \bar{P} \) for a given \( N_p \) value. However, for multiphase flow in saturated reservoirs, the average reservoir pressure \( \bar{P} \) does not appear explicitly in the MBE, rather, it is implicitly present in the various pressure-dependent PVT parameters in the MBE (i.e. \( B_o, B_g \) and \( R_s \)). Furthermore, the MBE for saturated reservoirs features the cumulative GOR term, \( R_p \), a parameter related to \( N_p \); hence solving the MBE for saturated reservoirs typically involves cumbersome iterative schemes.

There exist in the literature various methodologies of solving the MBE with the purpose of predicting estimates of cumulative oil production, \( N_p \) versus the declining average reservoir pressure, \( \bar{P} \). The methodologies typically involve some multi-step iterations as the equation is solved for \( N_p \) or \( G_p \) at various arbitrarily-chosen pressure nodes.

The need to solve this implicit problem without the rigor of human guesswork is the motivation for this work. This current work therefore presents a simple and computationally-cheap method for solving the MBE for the purpose of predicting estimates of cumulative oil production, \( N_p \) versus the declining average reservoir pressure, \( \bar{P} \).

In this new solution methodology, the philosophy of solving the MBE is analytically founded on the equality of the Left Hand Side, LHS (fluid withdrawal terms) and the Right Hand Side, RHS (fluid expansion terms) of the MBE for saturated reservoirs, and the pressure that upholds the equality.

This new solution methodology has been applied to two reservoir models and has been found to yield performance predictions that compares excellently well with predictions obtained from numerical solution (simulation) of complex fluid flow equations.

Introduction

The foremost essence of reservoir engineering is arguably the prediction of reservoir performance. Performance prediction commences with the prediction of cumulative oil produced as a function of declining reservoir pressure; thereafter, the well production rates are also predicted as a function of
declining reservoir pressure. The two profiles are correlated to yield production profile as a function of time. While the well performance prediction is achieved using inflow performance relationships; prediction of cumulative oil produced is obtained from solutions to relevant material balance equations (MBE). In addition to this predictive use, the MBE is also deployed in analysis of past performance data. In the predictive application of the MBE, estimates of amount of fluid initially in place is obtained from alternative sources and used to predict cumulative volumes of fluid that would have been withdrawn for the reservoir pressure to drop from initial pressure to some arbitrarily chosen future pressure. Conversely, in the analysis application, past performance data (i.e. cumulative volumes withdrawn) are analysed to determine the amount of fluid initially in place and to history match reservoir pressures with known cumulative volume withdrawn. From the foregoing, it is obvious that the use of the MBE for analysis is somewhat an explicit problem as the known past performance data are merely substituted in the equation which is then solved for the amount of fluid initially in place or the pressure drop. However, the predictive application poses a greater challenge, especially for saturated reservoirs where multiphase condition prevails.

For under-saturated reservoir (single-phase flow), the MBE presents cumulative oil produced, N_p as an explicit function of pressure drawdown (P – P_wl); hence a direct solution of the MBE yields N_p value for any arbitrary average reservoir pressure, \( \bar{P} \). However, for multiphase flow in saturated reservoirs being considered in this work, the average reservoir pressure \( \bar{P} \) does not appear explicitly in the MBE, rather, it is implicitly present in the various pressure-dependent PVT parameters in the MBE (i.e. \( B_o \), \( B_g \) and \( R_s \)). Furthermore, the MBE for saturated reservoirs features the cumulative GOR term, \( R_p \), a parameter directly related to N_p; hence solving the MBE for saturated reservoirs typically involves finding means of predicting the GOR term using saturation equations and relative permeability ratio. Since both N_p and \( R_p \) are unknown, and solving for one requires the value of the other, the entire task of performance prediction for saturated reservoirs typically involves some cumbersome iterative schemes.

There exist in the literature various methodologies of solving the MBE with the purpose of predicting estimates of cumulative oil production, N_p versus the declining average reservoir pressure, \( \bar{P} \). The methodologies typically involve some multi-step iterations as the equation is solved for N_p or G_p at various arbitrarily-chosen pressure nodes. Tracy (1955) presented the MBE in a new form by rearranging and expressing it in terms of three PVT functions which he had defined. He then formulated a technique for solving the MBE by combining it with the equation of instantaneous GOR. However, the technique is iterative as it watches for the convergence of GOR.

Muskat (1945) presented the MBE in a differential form. The technique involved solving for the differential \( \frac{dS_o}{dP} \) both at the previous known pressure and at the arbitrarily-chosen new lower pressure. The average of the two differential values is employed in determining the \( S_o \) at the new pressure and consequently, N_p at the new pressure is calculated with the new \( S_o \). Although the Muskat technique is not iterative, it involves some numerical differentiation. Turner’s (1944) technique is founded upon the calculation of G_p using a guessed N_p in two independent equations – the MBE, and the instantaneous GOR equation. If the two \( G_p \) values agree to a reasonable extent, the guessed N_p is taken to be correct, otherwise the procedure repeated with another guess of N_p. Expectedly, the iterations may make this technique rather cumbersome.

Ahmed and McKinney (2005) suggested a graphical technique for estimating average reservoir pressures corresponding to a known N_p value if a volumetric estimate of initial oil-in-place is available. Frederick and Kelkar (2005) formulated a technique for calculating average reservoir pressure for a given N_p; however, the technique is iterative as it starts off by making a guess of the ultimate recovery, \( N_{pf} \) at a given abandonment pressure. The technique is then implemented at each time node present in the production data (N_p versus t) and a consistency check is performed on the guessed \( N_{pf} \). The iterative
technique is repeated with another guess of \( N_{pf} \), if the last iteration is found to be inconsistent. Also, the technique requires the value of the well flowing pressure \( P_{wf} \).

The need to solve this implicit problem without the rigor of human guesswork is the motivation for this work. This current work therefore presents a simple and computationally-cheap method for solving the MBE for the purpose of predicting estimates of cumulative oil production, \( N_p \) versus the declining average reservoir pressure, \( P \). Although, detailed performance prediction is always possible using complex multi-dimensional, multiblock reservoir simulation models; however, this option is often expensive and time-consuming. Solution to zero-dimensional MBE tank model offers a reasonable approximate method for reservoir performance prediction; hence, one of the objectives of this work is to compare the reservoir performance profiles generated from MBE using the proposed solution methodology to those generated from a commercial reservoir simulator based on solution to complex flow equations.

**The Proposed Solution Methodology**

In this new solution methodology, the philosophy of solving the MBE is analytically founded on the equality of the Left Hand Side, \( \text{LHS} \) (fluid withdrawal terms) and the Right Hand Side, \( \text{RHS} \) (fluid expansion terms) of the MBE for saturated reservoirs. We recognized that for the equality to hold true, the difference between the LHS and RHS terms must be approximately zero. The MBE for a volumetric solution-gas-drive reservoir is presented in Ahmed and McKinney (2005) thus:

\[
N_p \left[ B_o + (R_p - R_s)B_g \right] = N \left[ (B_o - B_{oi}) + (R_{si} - R_s)B_g \right] 
\]

\( \text{LHS (fluid withdrawal terms) = RHS (fluid expansion terms)} \) \hspace{1cm} (1)

In developing this solution methodology, we simply transformed the MBE into a most basic format in which it became an equation only in \( N_p \), and \( N_{p, i} \). The transformation was made possible by first replacing each PVT properties (\( B_o \), \( B_g \), and \( R_s \)) in the MBE with its regressed function of pressure; i.e. \( B_o(P) ; B_g(P) \); and \( R_s(P) \). The regressed PVT functions were generated by curve-fitting the lab-determined PVT data of the reservoir fluid.

In handling the cumulative GOR term, \( G_{p, i} \), being the ratio of cumulative gas produced, \( G_p \), to cumulative oil produced, \( N_p \), we reckoned that \( G_p \) is the integral of the instantaneous GOR thus:

\[
G_{OR} = \frac{dG_p}{dN_p} 
\]

\( \text{Hence } G_p = \int_0^{N_p} G_{OR} dN_p \) \hspace{1cm} (2)

The integral in Equation 2 is approximated as the area under the curve of instantaneous GOR versus \( N_p \). The area is estimated using trapezoidal rule to yield the expression of \( G_p \) thus:

\[
G_p = \frac{1}{2} (G_{OR_{initial}} + G_{OR_{current}})(N_p - 0) 
\] \hspace{1cm} (3)

In the Equation 3, the \( G_{OR_{initial}} \) is the instantaneous GOR at the initial pressure of the reservoir; since the reservoir is saturated; i.e. it exists initially at its bubble point pressure, this term is taken as the initial solution gas oil ratio, \( R_{si} \), as there would be no free gas in the reservoir at this initial condition. The \( G_{OR_{current}} \) is the instantaneous GOR at the current reservoir pressure; the expression for the term is the GOR equation for solution-gas drive reservoirs. Thus the two GOR terms in Equation 3 are replaced thus:

\[
G_{OR_{initial}} = R_{si} \]

\[
G_{OR_{current}} = R_s + \frac{k_{rg}}{k_{ro}} \left( \frac{\mu_o B_o}{\mu_g B_g} \right) 
\]
The $R_p$ term in the LHS of the MBE is thus replaced by the expression in Equation 5. This expression features relative permeability terms, PVT and viscosity parameters. Each PVT and viscosity parameter is replaced with its regressed function of pressure. Typically, relative permeability data is presented as a plot relative permeability versus saturation; hence we replaced the relative permeability parameters with curve-fit saturations functions of SCAL data. Ultimately, the saturation terms in the resulting relative permeability curve-fit expressions were replaced with expressions relating saturations to Np thus:

\[
\frac{S_o}{N_p} = (1-S_{wi}) \left(1-N_p \left(\frac{B_o}{B_{oil}}\right)\right)
\]

\[
\frac{S_g}{N_p} = (1-S_{wi} - S_o)
\]

Upon implementing these series of transformations, the MBE became an equation with only $N_p$ and $\bar{p}$ as the two unknown parameters. Thus, the $R_p$ term is adequately handled and the MBE is transformed into an explicit function of pressure. Then we arranged the resulting equation in the form LHS – RHS = 0; that is the difference (error) between the LHS and the RHS terms is set to zero. This gave the final form of the MBE. We deploy this transformed MBE to two applications.

The foremost application is to predict reservoir performance, i.e. to predict the cumulative oil that would have been produced when reservoir pressure has dropped to a given pressure; this is accomplished by solving the transformed MBE equation for Np for desired pressure values. In this application, we reckon that for a given value of pressure, $\bar{p}$ substituted into the equation, any value of $N_p$ that forces the difference (error) between the LHS and the RHS terms to zero would certainly be the cumulative oil produced, $N_p$ that corresponds to that value of $\bar{p}$. Essentially, solving the equation for $N_p$ would yield the value of $N_p$ corresponding to the substituted $\bar{p}$. In solving the equation, we employed a simple spreadsheet tool that utilizes the fixed-point iteration algorithm. The tool varies the value in a spreadsheet cell until a formula in a second cell that depends on that value returns a desired outcome. We constituted a spreadsheet in which a guess of $N_p$ is entered into a cell from which it is called (along with the desired value) into other cells wherein fluid saturations ($S_o$ and $S_g$) are computed; the computed fluid saturations are then called into cells wherein the relative permeabilities are computed. Cells in which PVT and viscosity properties have been computed (using the regressed pressure functions) as well as the relative permeability cells are thereafter called into a cell wherein the cumulative gas-oil ratio, $R_p$ term is computed. The $R_p$ cell as well as the PVT properties cells are ultimately called into the two cells wherein the withdrawal term (LHS) and the expansion term (RHS) of the MBE are computed separately. Finally, the LHS cell and the RHS cell are called into a cell that holds the difference (error) between the two terms. Obviously, the difference (error) cell is fundamentally linked to the guessed $N_p$ cell; and therefore, the value in the difference cell would not be zero, since it evolved from a guessed $N_p$ value. At this point, the spreadsheet tool is employed to seek the $N_p$ value to be held in the $N_p$ cell in order to return zero in the difference cell. This ‘sought-out’ $N_p$ value is reckoned to be the $N_p$ value that validates the equality of the two sides (withdrawal and expansion sides) of the MBE, and hence, the $N_p$ value corresponding to the desired $\bar{p}$ value. This series of computations is repeated for all desired reservoir pressure $\bar{p}$ values, thereby generating the reservoir performance profile. A Visual Basic Macro was written to execute the entire scheme.

It is noticed that although the spreadsheet tool iterates internally, the iteration is devoid of the rigor of human guesswork and that this scheme works well with any initial guess to which the $N_p$ is set. It was
observed that there was no noticeable difference in the outcomes of the iterations when the initial guess (for all pressure nodes) was 0 STB and when the initial guess was the true (simulated) value of $N_p$.

In the second application, the equation was used to determine the average reservoir pressure to which the reservoir would have dropped as a consequence of producing a given $N_p$ of oil and $G_p$ of gas. This application finds usefulness and relevance in a reservoir permeability estimation technique recently formulated in Mosobalaje et al (2014). In this application, we considered that for a given value of $N_p$ and $G_p$, substituted into the equation, any value of $\bar{p}$ that forces the difference (error) to zero would certainly be the average reservoir pressure $\bar{p}$ that corresponds to that value of $N_p$.

The Procedure

The step-by-step procedure for implementing the proposed solution methodology is as follows:

1. Generate regressed functions of pressure for all PVT and viscosity parameters by curve-fitting laboratory-determined PVT and viscosity data. Fluid property correlations may also be used.
2. Choose an arbitrary value of average reservoir pressure $\bar{p}$ for which the cumulative oil produced $N_p$ is to be predicted. Evaluate the PVT and viscosity parameters at the chosen pressure using the regressed functions.
3. Using the PVT parameters evaluated in Step 2, compute the RHS of the MBE (the fluid expansion term) with Equation 8 below. Hold the computed RHS in a spreadsheet cell.

$$RHS = N \left( B_o - B_{oi} \right) + \left( R_{si} - R_s \right) B_g$$  \hspace{1cm} (8)

4. Make any guess of $N_p$; the value of this guess has no influence on accuracy of the solution methodology. Hold the guessed $N_p$ value in a spreadsheet cell.
5. Compute the oil saturation with the Equation 9 below by calling the guessed $N_p$ from the $N_p$ cell; use the evaluated $B_{oi}$ and $B_{oi}$ from Step 2 as well as estimates of initial oil in place and initial water saturation. Hold the computed oil saturation in a spreadsheet cell.

$$\overline{S_o} = (1 - S_{wi}) \left( 1 - \frac{N_p}{N} \right) \left( \frac{B_o}{B_{oi}} \right)$$  \hspace{1cm} (9)

6. Generate regressed functions of oil saturation for oil and gas relative permeabilities using SCAL-derived relative permeability data.
7. Compute oil and gas relative permeabilities with regressed functions generated in Step 6 by calling the saturation computed in Step 5, from the saturation cell. Hold each computed relative permeability in a spreadsheet cell.
8. Compute the cumulative gas-oil ratio term, $R_p$ with Equation 10 below, by calling the oil and gas relative permeabilities computed in Step 7, from the relative permeabilities cells and using the PVT and viscosity parameters evaluated in Step 2. Hold the computed $R_p$ in a spreadsheet.

$$R_p = \frac{1}{2} \left( R_{si} + R_s + \frac{k_{rg}}{k_{ro}} \left( \frac{\mu_o B_o}{\mu_g B_g} \right) \right)$$  \hspace{1cm} (10)

The $R_{si}$ parameter is typically available in lab-determined PVT data or could be obtained by substituting the initial reservoir pressure, $P_i$ into the regressed function for $R_s$ generated in Step 1.
9. Using the PVT parameters evaluated in Step 2 and by calling both the guessed $N_p$ and computed $R_p$ from their respective cells, compute the LHS of the MBE (the fluid withdrawal term) with Equation 11 below. Hold the computed LHS in a spreadsheet cell.

$$LHS = N_p \left[ B_o + (R_p - R_s) B_g \right]$$  \hspace{1cm} (11)

10. Compute the difference (error) between the LHS (from Step 9) and the RHS (from Step 3) of the MBE. Hold the difference in a cell.
11. Use any available spreadsheet optimization tool to find the value in the $N_p$ cell that returns a value
of zero for the difference (error) cell. That $N_p$ value is the true value corresponding to the chosen average reservoir pressure, $\bar{p}$.

12. Repeat Steps 2-5 and 7-12 for other chosen average reservoir pressures to obtain a forecast of reservoir performance profile.

**Application of the Methodology to Reservoir Models**

In order to verify the proposed solution methodology, the methodology has been applied to two reservoir models. The solution gas drive reservoir models published by Camacho and Raghavan (1987) and Frederick and Kelkar (2005) are essentially adopted here as Reservoir I and Reservoir II respectively. The reservoir properties, fluid PVT and the relative permeability data for Reservoir I are presented in Table 1 and Figures 1, 2 and 3.

### Table 1—Reservoir Properties

<table>
<thead>
<tr>
<th>Reservoir Properties</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drainage Radius, $r_e$ (ft)</td>
<td>2624.672</td>
</tr>
<tr>
<td>Porosity, $\phi$ (fraction)</td>
<td>0.3</td>
</tr>
<tr>
<td>Permeability, $K$ (mD)</td>
<td>10</td>
</tr>
<tr>
<td>Well Radius, $r_w$ (ft)</td>
<td>0.32808</td>
</tr>
<tr>
<td>Initial Reservoir Pressure, $P_i$ (psi)</td>
<td>5704.78</td>
</tr>
<tr>
<td>Skin Factor, $s$</td>
<td>10</td>
</tr>
<tr>
<td>Initial Water Saturation, $S_w$ (fraction)</td>
<td>0.3</td>
</tr>
<tr>
<td>Reservoir Temperature, $T_R$ (F)</td>
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</tr>
<tr>
<td>Gas Gravity, $\gamma_g$</td>
<td>0.65</td>
</tr>
<tr>
<td>Oil Gravity, $\gamma_o$ API</td>
<td>45.5</td>
</tr>
<tr>
<td>Reservoir Thickness, $h$ (ft)</td>
<td>15.55</td>
</tr>
<tr>
<td>STOIIP, $N$ (STB) (Volumetric Estimate)</td>
<td>7,015,933</td>
</tr>
<tr>
<td>Critical Bottom-hole Pressure, $P_{WFC}$ (psi)</td>
<td>1696</td>
</tr>
</tbody>
</table>

![Figure 1—Oil PVT Properties](image.jpg)
The lab-determined PVT and viscosity parameters were regressed to yield the following pressure functions for Reservoir I.

\[
B_o(P) = a_1P^3 + b_1P^2 + c_1P + d_1
\]

\[
R_o(P) = a_2P^3 + b_2P^2 + c_2P + d_2
\]

\[
B_g(P) = a_3P^{b_{bg}}
\]

\[
\mu_o(P) = a_4P^3 + b_4P^2 + c_4P + d_4
\]

\[
\mu_g(P) = a_5P^{b_{bg}}
\]

Similarly, lab-determined PVT and viscosity parameters were regressed to yield the following pressure functions for Reservoir II.
Table 2 lists the values of the coefficients of the regressed functions for both Reservoir I and Reservoir II.

\[
B_o(P) = a_1 P^3 + b_1 P^2 + c_1 P + d_1
\]

\[
R_s(P) = a_2 P^{b_2}
\]

\[
B_g(P) = a_3 P^{b_3}
\]

\[
\mu_o(P) = a_4 P^3 + b_4 P^2 + c_4 P + d_4
\]

\[
\mu_g(P) = a_5 P + b_5
\]

Results

Plots of reservoir performance profiles (i.e \(B_o\) versus \(P\)) generated using this new solution methodology for both Reservoir I and Reservoir II are shown in Figure 4 and Figure 5 respectively. In order to ascertain the accuracy of the solution scheme, the reservoir performance profile for each of the reservoir models were also determined using a commercial commercial reservoir simulator software (based on solution to complex multi-dimensional, multiblock reservoir simulation models and not on MBE) and these has been plotted along with profiles generated using this new solution methodology to the MBE. The two reservoir models have been originally simulated in Mosobalaje and Tiab (2013) where the relative permeability data was approximated as a straight line function due to the assumption of negligible capillary effect; hence the relative permeability data is treated same way here. Detailed specifications of the simulation are available in Mosobalaje and Tiab (2013); Mosobalaje and Tiab (2014) and Mosobalaje et al (2014).
From the plots, it is seen that the new solution methodology predicts performance profiles that closely match those predicted by the commercial reservoir simulator software. The average percent errors (deviation) recorded in Reservoir I and Reservoir II are 8.76% and 5.66% respectively. The deviation, although within acceptable range, has been found to be attributable to the approximation of the cumulative gas produced, \( R_p \) using (area under the curve of instantaneous GOR versus \( N_p \)) using the trapezoidal rule. The average percent deviation of the approximated \( R_p \) from the actual (simulated) \( R_p \) is 23% and 7.14%.
for Reservoir I and Reservoir II respectively. However, these deviations were offset by the accuracy of the regressed functions which replaced the PVT properties in the solution methodology.

**Illustrative Example**

For the purpose of illustration, pressure node $P = 5599.9$ psi was chosen to numerically demonstrate the application of the proposed methodology.

**Step 1:** Generating regressed function for PVT and viscosity parameters

The following were generated by curve-fitting lab-determined data (Figures 1 and 2).

$$
\begin{align*}
B_o &= -1.6 \times 10^{-12} P^3 + 2.8 \times 10^{-8} P^2 + 1.94 \times 10^{-5} P + 1.06 \\
R_s(P) &= 1.72 \times 10^{-9} P^3 + 8.5 \times 10^{-6} P^2 + 0.16 P - 6.09 \\
B_g(P) &= 6.113 P^{-1.1} \\
\mu_o(P) &= -2.48E^{-12} P^3 + 5.78E^{-8} P^2 + 4.49E^{-4} P + 1.44 \\
\mu_g(P) &= 3.43E^{-6} P + 0.0101
\end{align*}
$$

**Step 2:** Choice of average reservoir pressure $\bar{P}$

$\bar{P} = 5599.9$ psi. Substituting $\bar{P}$ into the regressed function in Step 1 yield the following

$$
\begin{align*}
B_o &= -1.55E^{-12}(5599.9^3) + 0.000000028(5599.9^2) + 1.94E^{-5}(5599.9) + 1.06 = 1.7745 \text{ RB/STB} \\
B_g(P) &= 6.113(5599.9)^{-1.1} = 0.000461 \text{ RB/STB} \\
R_s(P_{avg}) &= 1.72E^{-9}(5599.9)^3 + 8.5E^{-6}(5599.9)^2 + 0.16(5599.9) - 6.09 = 1458.5 \text{ Scf/STB} \\
\mu_o(P) &= -2.48E^{-12}(5599.9)^3 + 5.78E^{-8}(5599.9)^2 + 4.49E^{-4}(5599.9) + 1.44 = 0.302 \text{ cp} \\
\mu_g(P) &= 3.43E^{-6}(5599.9) + 0.0101 = 0.0293 \text{ cp}
\end{align*}
$$

**Step 3:** Computing the RHS of the MBE (the fluid expansion term)

$$
RHS = N \left( (B_o - B_o^i) + (R_s - R_s^i)B_g \right)
$$

$$
RHS = 7015933 \left( (1.7745 - 1.7941) + (1502.6 - 1458.5) \times 0.000461 \right)
$$

$$
RHS = 4778.61 \text{ RB}
$$

**Step 4:** Guess $N_p$

$$\quad N_{p guess} = 1 \text{ STB}$$

**Step 5:** Computing the oil saturation

$$
\bar{S}_o = (1 - S_{wi}) \left( 1 - \frac{N_p}{N} \right) \left( \frac{B_o}{B_o^i} \right)
$$

$$
\bar{S}_o = (1 - 0.3) \left( 1 - \frac{1}{7015933} \right) \left( \frac{1.7745}{1.794147237} \right) = 0.6923
$$

**Step 6:** Generating regressed functions for oil and gas relative permeabilities

$$
\begin{align*}
k_{ro} &= 1.067(\bar{S}_o) \\
k_{rg} &= 1.007(1 - \bar{S}_w - \bar{S}_o)
\end{align*}
$$

**Step 7:** Computing oil and gas relative permeabilities

$$
\begin{align*}
k_{ro}(\bar{S}_o) &= 1.067(0.6923) = 0.7387 \\
k_{rg}(\bar{S}_o) &= 1.007(1 - 0.3 - 0.6923) = 0.0077
\end{align*}
$$

**Step 8:** Computing the cumulative gas-oil ratio.

$$
\begin{align*}
R_p &= \frac{1}{2} \left[ R_{si} + R_s + k_{rg}(\frac{\mu_oB_o}{\mu_gB_g}) \right] \\
R_p &= \frac{1}{2} \left[ 1502.64 + 1458.5 + \frac{0.0077}{0.7387} \left( \frac{0.0077(1.7745)}{0.0293(0.000461)} \right) \right] = 1688.5 \text{ scf/STB}
\end{align*}
$$
**Step 9:** Computing the LHS of the MBE (the fluid withdrawal term).

\[ LHS = N_p [B_o + (R_p - R_s) B_g] \]
\[ LHS = 1[(1.7745) + (1688.5 - 1458.5) 0.000461] = 1.8804 \]

**Step 10:** Computing the difference between the LHS and the RHS

\[ 1.8804 - 4778.61 = -4774.7 \]

**Step 11:** The true \( N_p \) value

Using the spreadsheet tool yielded the following as the value of \( N_p \) that returns a zero in the difference cell

\[ N_{p\ true} = 2536.99 \ STB \]

Comparing \( N_p = 2536.99 \) to the actual (simulator) value \( N_p = 2430 \) yields only an error of 4.4%

**Summary and Conclusion**

This work has formulated a simple, accurate and computationally-cheap method for predicting reservoir performance in saturated reservoirs (solution-gas drive reservoirs) using a new solution methodology to the material balance equations of such reservoirs.

From the results presented in this work, the following conclusions are warranted.

1. The analytical solution of the Material Balance Equation is a computationally-cheap alternative to the numerical solution (simulation) of complex fluid flow equations, in attempts to predict reservoir performance profiles.
2. The solution to the material balance equation is essentially anchored on the equality of the fluid withdrawal term on one hand and the fluid expansion term on the other hand.
3. Each of the PVT and viscosity term present in the MBE could be replaced with its regressed pressure function, generated using lab-determined data.
4. The solution-gas drive MBE could be reduced to its most basic form by replacing each PVT parameter with its regressed function of pressure and by reducing the cumulative gas-oil ratio, \( R_p \) into a basic function of pressure and cumulative oil produced.
5. The solution-gas MBE in such basic form could be solved simply by ‘seeking’ for the \( N_p \) value that ‘forces’ an equality between the fluid withdrawal term and the fluid expansion term of the equation.
6. Although the solution algorithm may involve some intrinsic iterations (internal to the computing facility), and therefore requires that the \( N_p \) be set initially to a value; this scheme works well with any arbitrary initial value to which the \( N_p \) is set.
7. The error introduced by the approximation of the \( R_p \) term is attenuated by the accuracy of the regressed functions for PVT and viscosity data.
8. The cumulative oil produced, \( N_p \) from such MBE solution compares excellently well with simulation results.

**Acknowledgements**

Contributions from Dr. Orodu and Phebe Mosabalaje are acknowledged.

**Nomenclature**

- \( B \) formation volume factor RB/STB for liquid and RB/SCF for gas
- \( Gp \) Cumulative Gas Production, SCF
- \( k \) Absolute Permeability, mD
- \( Np \) Cumulative Oil Production, STB
- \( \bar{p} \) Average reservoir pressure, psi
- \( R_p, R_s \) Gas Oil Ratio, SCF/STB
So, Sg Saturations, fraction
µ viscosity, cp
φ porosity, fraction

SI METRIC CONVERSION FACTORS

<table>
<thead>
<tr>
<th>Conversion</th>
<th>SI Unit</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>bbl</td>
<td>m³</td>
<td>1.589873×10⁻³</td>
</tr>
<tr>
<td>cp</td>
<td>Pa·s</td>
<td>1.0×10⁻³</td>
</tr>
<tr>
<td>ft</td>
<td>m</td>
<td>3.048×10⁻¹</td>
</tr>
<tr>
<td>°F (°F – 32)/1.8</td>
<td>°C</td>
<td>5/9</td>
</tr>
<tr>
<td>mD</td>
<td>μm²</td>
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</tr>
<tr>
<td>psi</td>
<td>KPa</td>
<td>6.894757×10⁰</td>
</tr>
</tbody>
</table>

*Conversion Factor is exact.

References


